



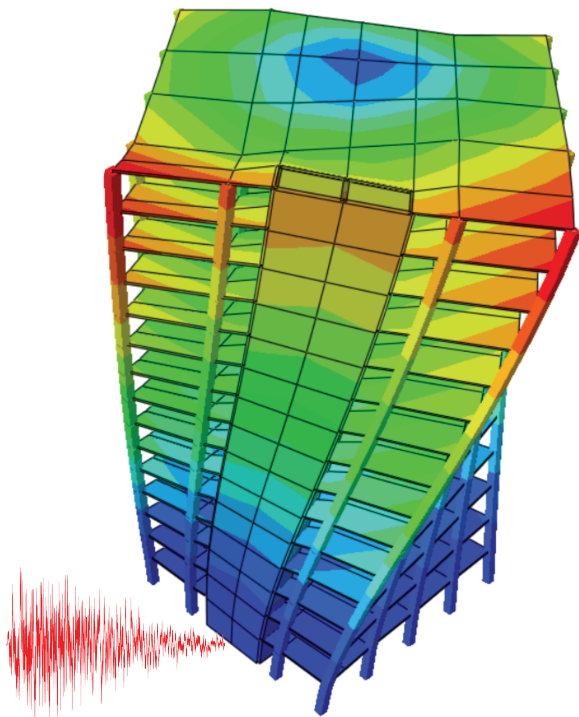
GALINDO
INGENIERÍA

SIMULATION OF SEISMIC GROUND MOTIONS FOR STRUCTURAL TIME-HISTORY ANALYSIS

FUNDAMENTALS, METHODOLOGY AND
THE EARTHQUAKE_2025 PROGRAM

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Chapter 1

Introduction

1.1 The Need for Time-History Analysis in Modern Seismic Engineering

Assessing the safety of structures against seismic action is one of the most complex challenges in civil engineering. Historically, modal response spectrum analysis has been the predominant method for earthquake-resistant design, providing a robust estimate of the maximum response demands for systems that behave in a predominantly linear manner. However, the evolution of seismic engineering towards performance-based design, coupled with the need to evaluate highly complex structures or those with advanced protection systems, has highlighted the fundamental inherent limitations of spectral methods.

Nonlinear behavior, cumulative fatigue damage, stiffness degradation, and the response of devices such as base isolators or energy dissipators are phenomena intrinsically dependent on the complete time history of the excitation. To capture this complexity, step-by-step analysis in the time domain is not merely an alternative but an imperative necessity. This type of analysis, however, faces a fundamental problem: the scarcity of real seismic records (accelerograms) that are representative of all possible design scenarios in terms of magnitude, fault distance, source mechanism, and, crucially, local soil conditions.

It is in this context that the generation of artificial ground motions assumes a leading role. For these generated earthquakes to be truly useful, they must satisfy a dual requirement. On one hand, they must be **statistically consistent** with the characteristics of the defined seismic hazard; this implies that physical parameters such as the duration of the shaking, the manner in which energy is released over time (the intensity envelope), or the total imparted energy (measured by the Arias Intensity) must be plausible and representative of the type of earthquake being modeled. On the other hand, the generated accelerograms must be **compatible with a design response spectrum**; a mathematical criterion that ensures the signal imposes on a set of idealized structures the same maximum demands that the code prescribes.

The ability to simultaneously meet the requirements of seismological realism and code compatibility is, therefore, the fundamental objective of advanced techniques for generating artificial earthquakes. This monograph addresses this challenge from its theoretical foundations to its implementation in a robust computational tool.

1.1.1 Artificial Earthquakes vs. Real Records: The Justification of the Method

While the use of real earthquake records can be a valid alternative, the generation of artificial earthquakes compatible with a design spectrum, as performed by the program `earthquake_2025`, offers a series of conceptual and practical advantages that make it the preferred methodology for state-of-the-art earthquake-resistant design. The fundamental reasons are:

- **The Design Spectrum as a Design Load vs. Real Load.** This is the most important conceptual argument. The design spectrum defined in a code does not represent an actual earthquake, but an idealized **seismic design load**. Just as an engineer does not design an office slab for the actual load on a specific day (the exact position of people, the weight and location of each piece of furniture at that instant, etc.), but uses a uniform design load (e.g., 2.5 kN/m^2) that safely envelops all possible loading configurations that will occur during the structure's service life, the design spectrum acts as a **statistical envelope** of the maximum demand of all possible earthquakes that could affect a site.

Using the record of an actual earthquake for analysis would be conceptually analogous to designing a slab based on a “photograph” of the loads acting on a random day in another building. It would be an arbitrary and partial representation of the true demand. Conversely, using an artificial earthquake whose spectrum is compatible with the code envelope is an extraordinarily efficient and robust way to verify structural performance against the full seismic hazard, as defined by the design criteria.

- **The Scarcity of Suitable Real Records.** It is virtually impossible to find a set of strong-motion records that simultaneously satisfy all specific project conditions: magnitude, distance, rupture mechanism, and, crucially, the same local soil conditions of the site. The use of artificial earthquakes removes this limitation by generating a tailored signal.
- **The Issue of Real Record Manipulation.** The common practice of scaling or modifying real records to match the target spectrum drastically alters the energy and frequency content of the signal, invalidating its realism. The generation of an artificial earthquake is a more transparent process: it creates a new stochastic realization that complies from its origin with the properties of the target spectrum.
- **Control and Parameterization for Analysis.** The use of artificial earthquakes provides the engineer with complete control over excitation parameters (duration, envelope shape, etc.), enabling sensitivity studies that are unattainable with a limited set of real records.
- **Absence of Instrumental Noise and Drift-Free Processing.** Artificial signals generated through rigorous stochastic formulations are inherently free of high-frequency instrumental noise, analog sensor saturation, and baseline drift problems that frequently plague historical records and require aggressive filtering prior to numerical integration.

Physical Limitations and Considerations in the Use of Synthetic Signals

Notwithstanding the undeniable advantages outlined above, sound engineering judgment also requires acknowledging the limitations inherent to stochastic generation and spectral matching methods:

- Complex Physical Ground Motion Phenomena.** Although a synthetic time history may achieve a near-perfect match with the target elastic spectrum, models based on harmonic superposition with random phases do not intrinsically reproduce certain nonlinear physical phenomena associated with the source and path. Notable among these are long-period velocity pulses associated with forward-directivity effects and permanent fling-step displacements, complex sedimentary basin resonances, or wave trains exhibiting strong phase correlation and dispersion typically observed in near-fault natural records.
- Dispersion Reduction in Nonlinear Analysis (*Spectral Matching*).** Several studies in contemporary earthquake engineering have pointed out that the use of individual accelerograms with excessively tight and uniform spectral matching tends to underestimate the natural dispersion of the inelastic structural response. By artificially removing the peaks and valleys naturally present in any real earthquake, the variability in engineering demand parameters (EDPs, such as peak interstory drifts or ductility demands) can be lower than that obtained from sets of scaled real records. Therefore, while they provide an impeccable representation of the code mean elastic demand, this aspect must be taken into account when interpreting statistical dispersion in advanced performance assessments.
- Code Perspective and Complementarity.** Modern international standards (such as ASCE 7 or the latest European codes) maintain an explicit preference for selected and scaled real records whenever a sufficient database of ground motions representative of the local seismotectonic environment is available. In this sense, the stochastic generation of signals using `earthquake_2025` should not be viewed as an exclusive substitute, but rather as an indispensable, top-tier complementary computational tool: whether for sites with scarce instrumental data, for verifications requiring strict spectral compatibility, or for conducting extensive parametric and fragility studies.

Corollary: The Need for Rigorous Compatibility. Since the artificial earthquake acts as the representative of the entire seismic hazard, the fidelity of this representation is of paramount importance. Poor compatibility, with significant valleys or peaks, implies that the signal does not properly represent the energy demand across certain frequency bands, which can lead to unreliable analysis results. For a structure whose fundamental period coincides with one of these mismatch regions, the analysis would yield unconservative results (if the computed spectrum falls below the target) or excessively conservative ones (if it lies above). In the context of Performance-Based Design, where precise damage assessment is sought, such discrepancies are unacceptable. Therefore, achieving “*super-compatibility*” is not a mere exercise in numerical precision, but a **fundamental requirement** to ensure that time-history analysis provides a robust verification of earthquake-resistant design.

Availability and Open-Source Philosophy. Both this monograph and the `earthquake_2025` software, along with all accompanying tools, are distributed free of charge (**MIT License**). They can be downloaded from <https://galindoingenieria.com/Downloads> and https://github.com/MarioGalindoQ/earthquake_2025. The objective is to provide the community of engineers, researchers, and students with a comprehensive and transparent set of tools under an open-source philosophy.

1.2 Objectives and Scope of this Monograph

This monograph pursues a dual objective. First, it aims to be a self-contained reference text that presents the fundamentals of seismology and seismic engineering necessary to understand the definition of earthquake action and its representation for structural analysis. Second, it seeks to serve as a technical manual and user guide for the `earthquake_2025` program, a new-generation computational tool for creating artificial earthquakes with a superior level of spectral compatibility.

The scope of the text has been structured to guide the reader from theoretical concepts to practical application:

- **Part I** establishes the theoretical framework. It explores the concepts of seismic characterization, the physical meaning of the response spectrum, the role of the Power Spectral Density Function (PSDF), and the intensity metrics that quantify the damage potential of a ground motion.
- **Part II** focuses on the methodology and application of the `earthquake_2025` program. It describes in detail the new **two-phase hybrid methodology** that allows for the generation of “*super-compatible*” accelerograms with a target spectrum. The first phase performs a robust adjustment in the frequency domain, while a novel second phase refines the signal in the time domain to achieve exceptional accuracy. A complete guide is provided on the use of the program and the interpretation of its results, which include:
 - The final time series of acceleration, velocity, and displacement of the optimized earthquake, with baseline correction.
 - A comparative analysis of the spectral error before and after the time-domain refinement phase.
 - Key engineering parameters such as PGA, PGV, PGD, Arias Intensity, and Significant Duration for the final earthquake.

Additionally, `process_earthquake_output_2025.py` is available, which is a post-processing tool embodied in a Python script designed to work in conjunction with the main program. This script fully automates the visualization and documentation of the results. By running it on one or more output files from `earthquake_2025`, the script creates a results folder for each one. Within it, it generates a complete set of high-quality plots (in PDF, PNG, SVG, and JPEG formats) and data files in CSV format.

These products include the response spectra (PSA, PSV, SD) compared to the target, and the final time histories of acceleration, velocity, and displacement, thus facilitating immediate visual analysis, the preparation of technical reports, and the export of data to other programs.

1.3 Structure of the Document

The monograph is organized into two distinct parts, followed by a set of appendices that provide supporting material and references.

Part I: Theoretical Foundations and Design Philosophy (Chapters 2 to 4), establishes the conceptual framework.

- **Chapter 2** explores the seismic phenomenon from the perspective of seismology, describing the source, wave propagation, and site effects.
- **Chapter 3** focuses on the characterization of ground motion for engineering purposes, defining the accelerogram, the PSDF, and intensity metrics.
- **Chapter 4** closes the theoretical part by connecting seismic action with the philosophy of earthquake-resistant design, explaining the importance of nonlinear response and performance-based design.

Part II: Methodology, Application, and Analysis Context (Chapters 5 to 9), concentrates on the computational tool and its ecosystem.

- **Chapter 5** serves as the main user guide for the `earthquake_2025` program, detailing the structure of the input file, providing a guide for interpreting the results, and offering a diagnostic manual for resolving convergence issues.
- **Chapter 6** unveils the internal computational methodology of `earthquake_2025`. It explains the two-phase algorithm: first, the signal synthesis and robust spectral adjustment in the frequency domain; second, the novel refinement phase in the time domain using corrective wavelets to achieve superior compatibility.
- **Chapter 7** presents the methods of structural analysis (MRSA, MTHA, IDIA, and EDIA), contextualizing where and why the generated signals are necessary.
- **Chapter 8** examines advanced seismic protection systems, demonstrating that their design demands the use of time-history analysis.
- **Chapter 9** offers the monograph's conclusions and proposes future lines of research.

Finally, the **Appendices** (A to E) provide essential reference material, including a dictionary of parameters, guides for the construction of code-based spectra, a complete case study, a glossary, and the bibliography.

1.4 Audience and Prior Knowledge

This text is primarily aimed at structural engineers, seismic engineers, researchers, and post-graduate students working in the field of structural dynamics and earthquake engineering. It is assumed that the reader is familiar with the fundamental concepts of structural dynamics, including the formulation of the equation of motion for single-degree-of-freedom (SDOF) systems and the principles of signal analysis. A basic knowledge of random processes theory will be beneficial for a deeper understanding of the methodology; for this purpose, texts with a practical approach such as that of Kay [36] are recommended.

1.5 Origin and Evolution of the Method: A Historical Context

It may be of interest to the reader to understand the context in which the method presented here was born and has evolved. The methodology was originally developed by the author in 1993 in a chapter of his doctoral thesis, titled “**Una Metodología Para el Análisis Numérico del Comportamiento Resistente No Lineal de Presas de Hormigón Bajo Cargas Estáticas y Dinámicas**” (A Methodology for the Numerical Analysis of the Nonlinear Resistant Behavior of Concrete Dams Under Static and Dynamic Loads) [26]. This work was part of a broader line of research on dam safety, also documented in the monographs [13] and [27]. As a result of the findings from this research, he was invited to participate as a co-author in the monograph “**IS-10 - Definición de la acción sísmica**” (IS-10 - Definition of the seismic action) [6], where he was responsible for writing Chapter 4, titled “**Acelerogramas artificiales compatibles con un espectro**” (Artificial accelerograms compatible with a spectrum). This chapter formally presents part of the methodology developed in the thesis.

Unfortunately, the original doctoral thesis is not available in digital repositories. However, the IS-10 monograph, which contains the conceptual foundation of the original method, can be freely downloaded and serves as an excellent reference for the theoretical starting point.

That said, the reader should bear in mind that three decades of research and computational development separate the original publication from the current version of the program. This monograph describes, in the manner of a manual, the operation of the **2025 version** of the `earthquake_2025` program. This new version not only refines the algorithms of its predecessors (1992, 2005, and 2022) but also introduces a completely new **two-phase hybrid methodology**, adding an optimization stage in the time domain to achieve an unprecedented level of spectral compatibility. Therefore, although the 1994 IS-10 monograph remains the conceptual reference, the 2025 source code and this text represent the **absolute source of truth** regarding the program’s behavior and capabilities.

On the Choice of Programming Language: It should be noted that the original version of the program was developed in strict Fortran 77. Although the author, in his current professional practice, programs almost exclusively in modern languages such as C++ and Python, the deliberate decision has been made to maintain `earthquake_2025` in Fortran 77.

This choice responds to a commitment to the research community, particularly in the field of finite element analysis, where Fortran remains a prevalent and highly important language. Maintaining compatibility with this ecosystem ensures that the program remains an accessible and relevant tool for a broad spectrum of the academic and professional community.

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Part I: Theoretical Foundations and Design Philosophy

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Chapter 2

The Seismic Phenomenon: From the Tectonic Source to the Site

2.1 The Seismic Source: The Origin of the Motion

2.1.1 Plate Tectonics and the Elastic Rebound Theory

The fundamental cause of the vast majority of destructive earthquakes lies in the dynamics of the Earth's lithosphere, described by the **theory of plate tectonics**. The lithosphere is fragmented into a series of rigid plates that move slowly over the asthenosphere, a more ductile layer of the upper mantle. At the boundaries of these plates, where they interact, enormous amounts of strain energy accumulate.

The physical mechanism for the release of this energy was brilliantly explained by H.F. Reid in 1910 [51], following his studies of the 1906 San Francisco earthquake. His **elastic rebound theory** postulates that tectonic forces subject the rocks to shear stress, which deforms them elastically along a fault. This deformation is slow and continuous over decades or centuries. When the accumulated stress in the rock exceeds the frictional strength of the fault, a sudden rupture occurs. The fault slips, and the rocks on both sides, suddenly released, elastically return (or “rebound”) to a configuration of lower strain. This violent release of stored strain energy is what generates seismic waves.

2.1.2 The Seismic Focus: Its Depth and Influence

The point within the Earth where the rupture process begins is called the **hypocenter** or focus. The focal depth is a first-order parameter that conditions the nature of the effects on the surface:

- **Shallow earthquakes (0-70 km):** These originate in the brittle Earth's crust. Being closer to the surface, the seismic wave energy has less distance to travel to reach structures, thus undergoing less attenuation. For this reason, the vast majority of destructive earthquakes are of the shallow type.
- **Intermediate (70-300 km) and deep (>300 km) earthquakes:** These originate within the oceanic plates subducting into the mantle. Although they can release enor-

mous amounts of energy, the waves must travel a great distance to the surface, causing significant attenuation of the high-frequency waves, which are the most damaging to stiff, low-rise structures. However, their energy can be felt over very large geographical areas.

Lightbulb Analogy: Magnitude vs. Intensity. To clarify the difference between two key terms that will be used throughout this text, a simple analogy is useful. The **magnitude** of an earthquake is like the intrinsic power of a lightbulb, for example, 100 watts. It is a single, invariable value that describes the total energy released at the source. **Intensity**, on the other hand, is like the brightness perceived at a specific point in a room (the structure’s site). This brightness depends on the lightbulb’s power (the magnitude), but also on the distance from it, the shadows cast by other objects in the room (e.g., mountain ranges), whether there is fog (ground attenuation), or if we are on a surface that reflects light (site amplification). Thus, a single lightbulb (a single earthquake of a fixed magnitude) will produce different levels of brightness (different intensities) at various points in the room (at different locations on the planet).

2.1.3 Magnitude: The Measure of Energy at the Source

Magnitude (M) is an instrumental and quantitative measure of the “size” of an earthquake, related to the amount of energy released at the focus. It is a unique and intrinsic value for each seismic event. The **Moment Magnitude Scale** (M_W) is the current global standard. It is based on the seismic moment ($M_0 = \mu AD$) and does not saturate for large earthquakes, faithfully reflecting the energy released [35]. Table 2.1 shows examples of magnitudes for historical earthquakes.

Table 2.1: Magnitudes (M_W) of Notable Historical Earthquakes.

Earthquake	Location	Magnitude (M_W)
Valdivia, 1960	Chile	9.5
Prince William Sound, 1964	Alaska, USA	9.2
Tōhoku, 2011	Japan	9.1
Sumatra-Andaman, 2004	Indian Ocean	9.1
Michoacán, 1985	Mexico	8.1
San Francisco, 1906	California, USA	7.9
Loma Prieta, 1989	California, USA	6.9
Kobe, 1995	Japan	6.9
Northridge, 1994	California, USA	6.7

2.1.4 Difference from the Richter Scale.

The general public commonly associates earthquake magnitude with the **Richter scale** (M_L) [31]. While it was a pioneering and historically significant scale, it is crucial to understand its limitations. The Richter scale is based on the maximum amplitude of waves recorded by a specific type of seismograph (Wood-Anderson) and was developed for the geological conditions of Southern California.

Its main drawback is the phenomenon of **saturation**: for large-magnitude earthquakes (typically above M_L 7), the instrument is unable to register the true increase in energy, so the scale “stalls” and systematically underestimates the actual size of the largest earthquakes.

The **Moment Magnitude Scale** (M_W), in contrast, does not measure an indirect wave amplitude but is calculated from the **seismic moment** ($M_0 = \mu AD$), a fundamental physical parameter that represents the total work done on the fault. Being directly related to the properties of the rupture source—the rock rigidity (μ), the area of the fault that slipped (A), and the average displacement on the fault (D)—the M_W scale does not saturate.

An additional advantage of its formulation is that it was intentionally calibrated so that its values would numerically coincide with those of previous scales (like the Richter scale) in the range of moderate magnitudes, approximately between 3 and 7. This allows the Moment Magnitude Scale to be seen not as a completely new system, but as a **robust and physically based extension** of the traditional scales.

For these reasons, the **Moment Magnitude Scale** (M_W) is the only one capable of consistently and accurately measuring the entire range of earthquakes, from the smallest to mega-earthquakes like the one in Chile in 1960 (M_W 9.5), and is the scientific and engineering standard used today.

2.2 Propagation of Seismic Waves

2.2.1 Wave Types and Arrival Sequence

Seismic energy travels in the form of waves. The sequence of arrival at a site is a key factor:

1. **P-waves (Primary)**: These are the fastest ($V_P \approx 5 - 8$ km/s in the crust). They arrive first and produce an initial, often vertical, jolt.
2. **S-waves (Secondary)**: These arrive next ($V_S \approx 3 - 5$ km/s). Their shearing motion is more destructive than that of P-waves. The time difference between the arrival of P- and S-waves allows for estimating the distance to the epicenter.
3. **Surface Waves (Love and Rayleigh)**: They are the last to arrive and often the most complex. They propagate at a speed slightly lower than that of S-waves (typically $\approx 90\%$ of V_S). Despite being the slowest, they usually have the largest amplitudes and the longest durations, making them the primary cause of structural damage, especially in far-field earthquakes.

Their high destructive potential is due to the nature of their motion, which is predominantly horizontal. **Love waves** induce a purely horizontal shearing motion (a “snaking”

of the ground perpendicular to the direction of propagation), while **Rayleigh waves** combine an elliptical vertical and horizontal motion. The dominant and most damaging component of this wave set is precisely the **large-amplitude horizontal shear**.

Herein lies a fundamental vulnerability of most building structures: they are designed and optimized with enormous stiffness and strength to withstand vertical gravity loads, but they are intrinsically much weaker and more flexible against large horizontal forces and displacements. The violent lateral “whipping” imposed by surface waves is what causes the greatest deformations in columns and walls, leading the structure to collapse.

This is the fundamental reason why, in the practice of seismic analysis, the excitation is almost always modeled using the horizontal components of the ground motion, as the vertical component, acting in the structure’s strong direction, is usually significantly less damaging to the overall response.

For a comprehensive treatment of wave propagation theory, reference texts such as [2] are recommended.

2.2.2 Duration and Attenuation: The Evolution of the Seismic Signal

The **duration of strong shaking** at a site depends on both the duration of the rupture at the source and the dispersion of the waves during their path. A large-magnitude earthquake ($M_W > 8$) involves a rupture that propagates along hundreds of kilometers, which can take several minutes and translates directly into a prolonged shaking at the site.

Simultaneously, the wave energy attenuates with distance, a phenomenon that reduces the shaking amplitude and modifies its character through two mechanisms:

- **Geometric Attenuation (*Geometric Spreading*):** As the waves travel away from the source, their energy is distributed over an ever-increasing wavefront. For body waves (P and S), the amplitude decays inversely proportional to the distance ($1/r$), while for surface waves, the decay is slower ($1/\sqrt{r}$).
- **Anelastic Attenuation (Material Damping):** This corresponds to the dissipation of energy as heat due to the internal friction of the rocks. This mechanism is highly frequency-dependent: it affects high frequencies much more pronouncedly, as they experience more damping cycles during propagation. The consequence is that the seismic signal not only loses amplitude with distance but is also “filtered,” losing its high-frequency content. This explains why distant earthquakes are characterized by a slow, long-period swaying motion, in contrast to the sharp, high-frequency shaking of nearby earthquakes.

2.3 Regional Seismicity and its Characteristics

Local tectonics dictate the typical characteristics of earthquakes, which are crucial for proper earthquake-resistant design.

Subduction Earthquakes (Chile, Peru, Pacific Coast of Japan): These earthquakes, generated by the sinking of an oceanic plate beneath a continental one, are the most powerful on the planet. They are characterized by:

- **Large Magnitudes and Durations:** They can exceed M_W 9.0, with ruptures lasting several minutes and generating very prolonged shaking.
- **Broad Frequency Content:** They produce a large amount of energy at both high and low frequencies. This makes them hazardous to all types of structures, from low-rise, stiff buildings (sensitive to high frequencies) to tall, flexible buildings (sensitive to low frequencies).
- **Coastal Location:** As they occur offshore, the effects on the coast are devastating, not only from the shaking but also from the associated risk of tsunamis.

Strike-Slip Fault Earthquakes (California, USA): Dominated by the San Andreas Fault system, these earthquakes are of the shallow type.

- **High-Frequency Content:** They are generally rich in high frequencies, primarily affecting low-rise, stiff structures.
- **Rupture Directivity:** At sites near the fault, they can generate long-period velocity pulses, which are very demanding for flexible structures like bridges or tall buildings.
- **Inland Location:** The hazard is distributed across the continent, directly affecting urban centers.

Complex Seismicity (Japan, Mexico):

- **Japan:** It has one of the most complex seismic environments in the world. It experiences both subduction mega-earthquakes off its Pacific coast (like the 2011 Tōhoku event), with characteristics similar to those in Chile, and very destructive shallow crustal earthquakes within its islands (like the 1995 Kobe event). This dual threat requires a design that considers an extremely wide range of frequencies.
- **Mexico:** The 1985 earthquake demonstrated that a subduction earthquake, occurring at a great distance on the coast, can be catastrophic inland if its low-frequency waves encounter soil conditions that amplify them through resonance, as happened in the valley of Mexico City.

2.4 Measuring the Effect: Intensity and Site Effects

2.4.1 The Modified Mercalli Intensity (MMI) Scale

Intensity describes the severity of shaking at a specific location. Table 2.2 presents a detailed description of the MMI Scale levels.

Table 2.2: Modified Mercalli Intensity (MMI) Scale - Detailed Description.

Level	Shaking	Description of Typical Effects
I	Not felt	Not felt except by a very few under especially favorable conditions.
II-III	Weak	Felt by persons at rest, especially on upper floors. Hanging objects may swing.
IV	Light	Felt indoors by many, outdoors by few. Sensation like heavy truck striking building. Windows, dishes, doors disturbed.
V	Moderate	Felt by nearly everyone. Overturning of small unstable objects and falling of objects from shelves. Doors swing, close, open.
VI	Strong	Felt by all; many frightened and run outdoors. Heavy furniture moved. Fallen plaster. Slight damage to masonry chimneys.
VII	Very Strong	Difficult to stand. Negligible damage in buildings of good seismic-resistant design. Moderate damage in ordinary well-built structures (cracks in walls). Considerable damage in vulnerable, unreinforced masonry structures.
VIII	Severe	Slight to moderate damage in earthquake-resistant structures (e.g., cracks in concrete columns). Considerable damage in ordinary buildings with partial collapse. Great damage or collapse of unreinforced masonry structures.
IX	Violent	General panic. Considerable damage in earthquake-resistant structures, with potential loss of verticality and partial collapse. Ordinary buildings severely damaged or collapsed. Great damage to foundations. Underground pipes broken.
X	Extreme	Collapse of many masonry and frame structures. Severe damage and possible collapse of well-built reinforced concrete structures. Steel structures may be permanently deformed. Rails bent. Landslides.
XI-XII	Cataclysmic	Virtually all structures destroyed. Even superior-designed steel and concrete structures suffer critical damage or collapse. Bridges destroyed. Large-scale ground failure. Waves seen on ground surface. Near-total destruction.

2.4.2 Site Effects: The Role of Local Soil and Topography

Two locations at the same distance from the same epicenter can experience drastically different levels of shaking. This phenomenon is due to **site effects**, the understanding of which is fundamental in seismic and geotechnical engineering [30]. Site effects encompass both the influence of the soil profile (1D site effects) and the shape of the terrain (2D/3D topographic effects).

Amplification by Soil Stratigraphy. When seismic waves travel from hard, dense bedrock to softer, less dense surface sediment deposits, their propagation speed decreases. To conserve energy, the amplitude of the waves must increase, a phenomenon known as **site amplification**, analogous to how ocean waves grow in height as they approach the coast.

More importantly, the soil deposit is not a uniform amplifier but a dynamic filter that extraordinarily selects and amplifies frequencies close to its own fundamental period of vibration. The 1985 Michoacán earthquake is the paradigmatic example of this effect: a subduction earthquake that occurred more than 350 km away generated moderate shaking on the coast but caused the collapse of hundreds of mid-rise buildings (between 5 and 15 stories) in Mexico City. The reason was that the long-period waves generated by the distant earthquake acted as the **exciting action**. Upon reaching the city's valley, they encountered the soft clay deposits of the ancient lakebed. The fundamental period of these deposits, around 2 seconds, coincided with that of the incoming seismic waves, causing a **resonance** phenomenon, which amplified the ground motion extraordinarily, precisely in the period range of the buildings that collapsed.

Topographic Amplification. The shape of the terrain also modifies the shaking. Hills, slopes, and canyons can focus or de-focus seismic energy. Typically, hilltops and ridge crests experience significant amplification of horizontal acceleration, as the waves become “trapped” and reflect, interfering constructively. Conversely, the base of valleys may experience de-amplification. This effect is critical for structures located on non-flat terrain.

The interaction of these effects defines the final character of the motion at the base of a structure:

- **Hard Ground (Rock):** The motion tends to have lower amplitude and be richer in high frequencies. It is more damaging to low-rise, stiff structures (masonry houses, nuclear reactors), whose short fundamental periods can lead to a resonant response.
- **Soft Ground (Clay, Sediments):** The motion has higher amplitude and longer duration. The soil tends to filter out high frequencies and amplify low ones, creating a quasi-harmonic excitation. It is devastating for tall, flexible structures (skyscrapers, long-span bridges), whose long periods can coincide with those of the ground, causing a resonant response.

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Chapter 3

Characterization of Ground Motion for Engineering

3.1 The Accelerogram: The “Fingerprint” at the Site

The conceptual bridge between seismology and structural engineering is the **accelerogram**. For the engineer, the complex cascade of phenomena that begins at the tectonic source and propagates through the crust materializes into a single function of time: the ground acceleration at the base of the structure, $a_g(t)$. This record is not a mere abstraction but the complete and fundamental description of the dynamic load to which the structure will be subjected. It is the “fingerprint” of the earthquake at that specific point, containing the combined signature of the source, the path, and the local site effects.

A complete seismic record, obtained by an accelerograph, consists of three orthogonal components of acceleration (one vertical and two horizontal). From these records, engineering has traditionally defined a set of **peak parameters**, which offer an initial estimate of the motion’s severity:

- **PGA (Peak Ground Acceleration):** This is the maximum absolute value of the accelerogram, $\max |a_g(t)|$. Although a complete seismic record has three components, in the context of seismic engineering and this monograph, the term PGA conventionally refers to the maximum value of one of the two horizontal components.

For decades, it was the primary parameter for characterizing seismic hazard and anchoring design spectra. However, research has shown that its correlation with structural damage is, in many cases, poor. A high PGA can result from a single, very high-frequency (very short-period) pulse, which possesses little energy and is not capable of significantly exciting most civil engineering structures, whose fundamental periods are longer.

- **PGV (Peak Ground Velocity):** This is the maximum absolute value of the ground velocity, $\max |v_g(t)|$, obtained by time integration of the acceleration record. As with PGA, this term conventionally refers to the horizontal component of motion. PGV has proven to be a much more robust indicator of damage potential, especially for structures with intermediate periods (approximately between 0.5 and 2.5 seconds). It correlates well with the strain energy that the structure must absorb, as the imparted kinetic energy is proportional to the velocity squared.
- **PGD (Peak Ground Displacement):** This is the maximum absolute value of the ground displacement, $\max |d_g(t)|$, obtained by double integration of the accelerogram. This parameter, also referring to the horizontal component, is crucial for the design of very long-period structural systems, such as long-span bridges, base-isolated buildings (where the displacement of the isolation system is a limiting factor), and for the interaction of buried structures (pipelines, tunnels) with the surrounding ground.

It is imperative to note that obtaining reliable velocity and displacement records from an accelerogram is non-trivial. The integration process is highly sensitive to the low-frequency noise inherent in the digitized signal. This noise, although of small amplitude in the acceleration, is greatly magnified with each integration, producing unrealistic drifts in the velocity and, especially, displacement time series. Therefore, it is essential to apply a rigorous **signal processing**, which includes **filtering** and **baseline correction**, before these integrated records can be used. This topic is addressed in detail in Chapter 6 of this monograph from the perspective of artificial earthquake generation, when describing the algorithm implemented in `earthquake_2025`.

3.2 Site Effects: Amplification by Geotechnical Conditions

3.2.1 The Physical Mechanism: Impedance Contrast

One of the most determining phenomena in seismic engineering is the modification of ground motion by local geological conditions, a phenomenon known as **site effects** [30]. The underlying physics is explained by the principle of energy conservation and the contrast between the dynamic properties of materials, a concept quantified by seismic impedance.

Definition of Seismic Impedance. The **seismic impedance** of a material, I_s , is an intrinsic measure of its resistance to being set in motion by a passing wave. It is calculated as the product of the material's density, ρ , and the shear wave propagation velocity, V_S :

$$I_s = \rho \cdot V_S$$

The shear wave velocity, in turn, depends directly on the material's stiffness (G , the shear modulus) and its density, according to the relation:

$$V_S = \sqrt{\frac{G}{\rho}}$$

Intuitively, a material with high impedance (like rock, which is very stiff and dense, with high G and V_S) strongly resists being deformed. A low-impedance material (like clay, which is soft and less dense, with low G and V_S) offers much less resistance.

Impedance Contrast and Amplitude Amplification. The amplification mechanism is best understood with an analogy: a wave traveling along a heavy, thick rope (high impedance) that is tied to a light, thin rope (low impedance). For energy to be transmitted from the heavy rope to the light one, the latter must move with a much larger amplitude.

The same principle applies to soil: when a seismic wavefront propagates from bedrock (high impedance) into a surface deposit of soft soil (low impedance), the energy carried by the wave must, to a large extent, be conserved. As the propagation velocity decreases drastically in the softer medium, the amplitude of the motion must increase to maintain the energy flux. This effect is responsible for sites on sedimentary deposits experiencing surface accelerations several times higher than those on a nearby rock outcrop.

3.2.2 Response on Rock vs. Soft Soil and the Fundamental Period

The phenomenon of resonance occurs when the fundamental period of vibration of a structure, T_e , coincides with the dominant period of the seismic excitation affecting it. It is crucial to understand that the load acting on the base of the structure is not the original motion in the rock, but the ground motion at the surface, which has been modified by the soil deposit itself. The soil deposit is not a simple uniform amplifier but a dynamic filter that selectively and extraordinarily amplifies frequencies close to its own fundamental period of vibration, T_s .

The Fundamental Period of the Soil (T_s). For a uniform soil layer of thickness H over a much stiffer bedrock, the fundamental period can be approximated as:

$$T_s \approx \frac{4H}{V_S}$$

This fundamental formula in geotechnical earthquake engineering is derived from the behavior of shear waves trapped in the layer. A wave ascending from the rock reflects off the free surface (where stress is zero), travels downward, reflects again at the bedrock contact, and travels back up. The first mode of vibration (the fundamental) corresponds to a standing wave whose wavelength, λ , is four times the layer thickness ($\lambda = 4H$). Since the period is the wavelength divided by the propagation velocity, it follows directly that $T_s = \lambda/V_S = 4H/V_S$. This clearly shows that the soil period depends directly on both its thickness (depth) and its stiffness (implicit in V_S).

Consequences for Design: The Danger of Soil-Structure Resonance. Seismic hazard is drastically magnified when an “amplification cascade” or **double resonance** occurs. This phenomenon occurs when the fundamental period of the structure (T_e) coincides with the fundamental period of the soil deposit on which it rests (T_s). The structure does not respond directly to the motion in the bedrock, but to the motion at the surface, which has already been filtered and amplified by the soil. If the structure has a period similar to that of the soil, it enters into resonance with an excitation that is already strongly energized at that same frequency, producing a potentially catastrophic response. The fundamental objective of good geotechnical and structural design is, therefore, to avoid the condition $T_e \approx T_s$.

- **Rock or Very Stiff Soil Sites ($V_{S,30} > 760$ m/s):** These sites are characterized by a very high shear wave velocity V_S , resulting in an extremely short site fundamental period, T_s (close to zero). The surface motion is a sharp, short-duration shaking rich in high frequencies (a “whiplash” effect). This type of excitation transmits large accelerations and is particularly damaging to very stiff, short-period structures ($T_e < 0.5$ s), such as low-rise masonry buildings, gravity dams, or industrial and nuclear equipment. If the period of these structures coincides with the high-frequency range of the excitation, resonance induces very high inertial forces ($F = ma$) that can cause brittle shear failures.
- **Soft Soil Sites ($V_{S,30} < 180$ m/s):** A deep, soft soil deposit has a low shear wave velocity V_S , resulting in a long fundamental period T_s (typically > 1.0 s). The deposit acts as a band-pass filter: it attenuates the high frequencies of the rock motion and extraordinarily amplifies a narrow range of low frequencies around its period T_s . The resulting surface motion is a slow, large-amplitude, long-duration, and quasi-harmonic sway. This scenario is most dangerous for tall, flexible structures (skyscrapers, cable-stayed bridges, elevated tanks), whose fundamental periods T_e are also long. The coincidence of periods ($T_e \approx T_s$) induces massive lateral displacements that can lead to collapse due to global instability (P-Delta effects) or by exceeding the deformation capacity of the elements.

Given the critical importance of this interaction phenomenon, modern seismic codes have abandoned the concept of a single design spectrum. Instead, they establish a geotechnical site classification based on the average shear wave velocity in the upper 30 meters of the subsoil, $V_{S,30}$. This forces the engineer to design the structure not against a generic hazard, but against the specific hazard as modified by the local conditions of its site.

3.3 Instrumental Measures of Motion Intensity

3.3.1 Arias Intensity (AI)

Defined in 1970 by Professor Arturo Arias [3], I_A is an integral measure of the total energy per unit mass imparted by the earthquake. Its formulation:

$$I_A = (\pi/2g) \int_0^{T_d} [a_g(t)]^2 dt$$

makes it a much more reliable indicator of destructive potential than peak parameters, as it considers the amplitude, frequency content, and duration of the shaking. It is especially relevant for evaluating cumulative phenomena. The new `earthquake_2025` program calculates this intensity for each artificial earthquake it generates.

Table 3.1 shows an empirical correlation between I_A values and MMI levels, providing a valuable link between the instrumental measure and observed damage.

Table 3.1: Approximate Correlation between Arias Intensity (I_A) and the Modified Mercalli Intensity (MMI) Scale.

Arias Intensity		Severity Level	Approximate MMI Correlation
[m/s]	[cm/s]		
< 0.1	< 10	Weak Shaking	I-IV
0.1 - 0.5	10 - 50	Light to Moderate Shaking	V-VI
0.5 - 1.5	50 - 150	Strong Shaking	VII
1.5 - 4.0	150 - 400	Very Strong Shaking	VIII
> 4.0	> 400	Severe to Extreme Shaking	IX+

Note: These ranges are a general guide. Actual damage is critically dependent on the specific vulnerability of structures and local conditions.

3.3.2 Significant Duration (D5-95)

Significant Duration quantifies the intense shaking phase of the motion. Defined by Trifunac and Brady (1975) as the time interval during which the bulk of the energy is released — specifically, between 5% and 95% of the total Arias Intensity value ($D_{5-95} = t_{95\%I_A} - t_{5\%I_A}$) — this metric is of vital importance because, unlike peak parameters such as PGA, it captures the **exposure time** of the structure to the most destructive phase of the earthquake [58].

A long-duration earthquake, even with a moderate PGA, can be much more damaging than a high-PGA but very short earthquake. Its relevance is particularly critical in the analysis of phenomena where damage is cumulative and depends on the number of significant loading cycles. Table 3.2 details its impact in various areas of engineering.

The `earthquake_2025` program calculates this duration for each artificial earthquake it generates, allowing the engineer to assess whether the motion's characteristics are appropriate for the type of analysis to be performed.

Table 3.2: Applications and Relevance of Significant Duration (D_{5-95}).

Engineering Field	Influenced Phenomenon	Mechanism and Consequences
Structural Engineering	Strength and Stiffness Degradation (Reinforced Concrete)	Multiple cycles of inelastic excursion cause crack opening and closing, cover spalling, and degradation of reinforcement bond. Longer duration leads to greater accumulated damage.
	Low-Cycle Fatigue (Structural Steel)	Welded connections and steel members are susceptible to fatigue fracture after a relatively low number of high-strain cycles. A prolonged duration increases the number of these cycles, raising the risk of failure.
Geotechnical Engineering	Liquefaction Potential (Saturated Granular Soils)	This is the quintessential example. The buildup of pore water pressure in soil is a cumulative process. Each seismic shear cycle increases the pressure. A longer duration means more cycles, drastically increasing the probability that pore pressure will equal the effective stress, causing liquefaction.
	Slope and Embankment Stability	Soil strength can degrade under cyclic loading. Prolonged shaking can cause cumulative permanent displacements in slopes and embankments, potentially leading to failure.
Seismic Protection Systems	Heating in Viscous and Yielding Devices	Energy dissipators convert kinetic energy into heat. A long duration of shaking can cause overheating of the devices, altering their mechanical properties and reducing their effectiveness.
	Displacement Accumulation (“Ratcheting”)	In isolation systems or structures with asymmetric behavior, a long duration can lead to a progressive accumulation of displacement in one direction, a phenomenon known as “ratcheting”, which can exceed the system’s deformation capacity.

3.3.3 Cumulative Absolute Velocity (CAV)

Cumulative Absolute Velocity (CAV) is an intensity measure developed by the EPRI (Electric Power Research Institute) for seismic hazard assessment of nuclear power plants, which has shown an excellent correlation with the onset of structural damage across a wide range of structures. Unlike PGA, which represents a single instant and can be misleading, CAV integrates the absolute value of acceleration over the entire duration of the record:

$$\text{CAV} = \int_0^{T_d} |a_g(t)| dt$$

The result, with units of velocity (e.g., m/s), is a robust measure of the total shaking. By considering both the amplitude and the duration of the strong motion phase, CAV is much less sensitive than PGA to isolated high-frequency spikes that have little energy and do not cause significant damage.

Note on Terminology. Although its name, “Cumulative Absolute Velocity”, might suggest an integration of velocity, the metric is defined, as shown in the equation above, from the integral of the **acceleration**. The name derives from the dimensional result of the operation: the integral of acceleration (L/T²) over time (T) yields a quantity with units of velocity (L/T).

The primary utility of CAV lies in its ability to define a “non-damaging shaking” threshold. Numerous studies on real earthquake records have shown that ground motions with a CAV below a certain value rarely produce significant structural damage, regardless of their PGA. This makes it a very powerful diagnostic tool in the post-earthquake assessment of facilities.

Table 3.3 presents an empirical and approximate correlation between CAV values and the levels of the Modified Mercalli Intensity (MMI) Scale, providing the engineer with a practical guide to interpret the severity of a motion from its CAV value. The `earthquake_2025` program calculates this important parameter for each generated artificial earthquake.

Table 3.3: Approximate Correlation between Cumulative Absolute Velocity (CAV) and the Modified Mercalli Intensity (MMI) Scale.

CAV		Severity Level	Approximate MMI Correlation
[m/s]	[cm/s]		
< 0.2	< 20	Weak Shaking	I-IV
0.2 - 1.0	20 - 100	Light Shaking	V
1.0 - 4.0	100 - 400	Moderate to Strong Shaking	VI-VII
> 4.0	> 400	Very Strong to Severe Shaking	VIII+

Note: These ranges are a general guide and have been compiled from various sources in the seismic engineering literature. The actual damage threshold is highly dependent on the vulnerability of each structure.

3.3.4 Potential Destructive Impulse (ID)

The Potential Destructive Impulse, or I_D , is an intensity measure proposed by Araya and Saragoni that is particularly sensitive to asymmetry and long-duration velocity pulses in a seismic motion. While Arias Intensity (I_A) quantifies the total imparted energy (an always-positive value), I_D measures the **net directional energy**.

Its original formulation is:

$$I_D = \frac{\pi}{2g} \int_0^{T_d} a_g^2(t) \cdot \text{sgn}(v_g(t)) dt$$

where $\text{sgn}(v_g(t))$ is the sign function of the ground velocity (+1 if velocity is positive, -1 if negative). The integral accumulates energy (a_g^2) positively during positive velocity cycles but subtracts it during negative velocity cycles.

For greater diagnostic clarity, the `earthquake_2025` program calculates and reports the two components of this integral separately:

- **Positive Destructive Impulse (I_D^+):** The energy imparted during positive velocity cycles.
- **Negative Destructive Impulse (I_D^-):** The energy imparted during negative velocity cycles.

The sum of both components ($I_D^+ + I_D^-$) is, by definition, equal to the Arias Intensity (I_A), serving as a check on the calculation's consistency. The difference ($I_D^+ - I_D^-$) corresponds to the total I_D and quantifies the net energy impulse.

Interpretation of Numerical Values. The true value of I_D lies not in its absolute magnitude, but in its **relationship with Arias Intensity**. This comparison allows for classifying the character of the motion:

- **Oscillatory / Symmetric Earthquakes:** A motion with many loading cycles of similar amplitude in both directions (such as far-field earthquakes or **artificial earthquakes generated with random phases**) will have very similar values of I_D^+ and I_D^- . For example, if $I_A = 2.0$ m/s, one might expect $I_D^+ \approx 1.0$ m/s and $I_D^- \approx 1.0$ m/s. In this case, the net impulse ($I_D^+ - I_D^-$) will be close to zero, indicating the absence of a dominant velocity pulse.
- **Impulsive / Asymmetric Earthquakes:** A near-field motion affected by directivity effects (“fling-step”) will contain a long-duration velocity pulse in one direction. This will cause one of the I_D components to be much larger than the other. For example, for an $I_A = 2.0$ m/s, one might find that $I_D^+ = 1.8$ m/s and $I_D^- = 0.2$ m/s. The net impulse ($I_D^+ - I_D^-$) would be 1.6 m/s, a very high value on the same order of magnitude as I_A itself, which is a clear indicator of an impulsive and potentially very damaging motion for flexible structures.

Therefore, the inclusion of these metrics in the output of `earthquake_2025` not only allows the user to verify the expected symmetry in an artificial earthquake but also provides a tool to analyze and compare it with records of real earthquakes that may contain pulses.

3.3.5 Housner's Spectral Intensity (HI)

Housner's Spectral Intensity, or HI, is a measure of an earthquake's damage potential that focuses directly on the response of structures rather than on the intrinsic properties of the ground motion [33]. Its conception is based on the premise that the Pseudo-Velocity Spectrum (*PSV*) is directly related to the strain energy that a structure must dissipate.

HI is defined as the integral of the Pseudo-Velocity Spectrum (*PSV*) for a fixed damping over a range of periods. In the standard definition, HI is calculated for 5% damping over an integration range of 0.1 to 2.5 seconds:

$$HI_{5\%} = \int_{0.1s}^{2.5s} PSV(T, \xi = 0.05) dT$$

The result, with units of length (e.g., cm or m), represents the area under the pseudo-velocity spectrum curve in the period range most relevant to structural engineering. Unlike Arias Intensity, which is very sensitive to total duration and high-frequency content, Housner's Intensity is often a better indicator of the deformation demand on a set of buildings with varied fundamental periods. A high HI value indicates that the earthquake is particularly severe for conventional structures.

To provide a practical context for the numerical values, Table 3.4 shows an approximate correlation between Housner's Intensity and expected damage levels, linked to the Modified Mercalli Intensity (MMI) Scale.

Table 3.4: Approximate Correlation between Housner's Intensity ($HI_{5\%}$) and the Modified Mercalli Intensity (MMI) Scale.

Housner's Intensity ($HI_{5\%}$)		Severity Level	Approximate MMI Correlation
[m]	[cm]		
< 0.1	< 10	Weak to Light Shaking	IV-V
0.1 - 0.3	10 - 30	Moderate Shaking	VI
0.3 - 0.8	30 - 80	Strong Shaking	VII
0.8 - 2.0	80 - 200	Very Strong Shaking	VIII
> 2.0	> 200	Severe to Extreme Shaking	IX+

Note: These ranges are an empirical and approximate guide. Housner's Intensity does not capture the effects of earthquake duration, which can be critical for the final damage.

The `earthquake_2025` program calculates this metric using the response spectrum of the **primary damping** specified by the user in the input file. It is vitally important for the user to note the following: the correlation with damage levels shown in Table 3.4 is established for the standard definition of Housner's Intensity, which assumes 5% damping. If the user specifies a primary damping different from 0.05, the HI value reported by the program will be mathematically correct for that damping level, but its direct interpretation using the correlation table will no longer be valid.

3.3.6 Peak Floor Acceleration (PFA)

While PGA describes the acceleration at the base of the structure, Peak Floor Acceleration (PFA) quantifies the maximum absolute acceleration experienced within it. As a building vibrates, its upper floors can experience accelerations significantly greater than those of the ground, a phenomenon known as dynamic amplification. PFA is the **critical intensity measure for the design of non-structural components** and their contents (mechanical equipment, facades, installations, etc.).

PFA is not a single value but varies with height and the dynamic properties of the structure. To characterize this demand generally, the `earthquake_2025` program calculates the **Absolute Acceleration Response Spectrum**. This is obtained by calculating, for each period T_n of the spectrum, the maximum absolute acceleration, $\max_t |\ddot{u}(t) + a_g(t)|$, of an SDOF oscillator subjected to the earthquake. The result is an envelope of the maximum acceleration that any single-degree-of-freedom non-structural component would experience, as a function of its own vibration period.

Relationship with the Pseudo-Acceleration Spectrum (PSA). It is important to note that the PFA Spectrum is, by definition, identical to the Absolute Acceleration Spectrum ($S_{a,abs}$). In turn, this is mathematically very close to the Pseudo-Acceleration Spectrum ($PSA = \omega_n^2 \cdot SD$) for low damping levels. The approximation $S_{a,abs} \approx PSA$ is excellent for viscous damping values ξ below approximately 10%. However, as damping increases, damping forces become more relevant in the dynamic response, and the difference between the actual absolute acceleration and the pseudo-acceleration can become significant. The `earthquake_2025` program calculates the exact value of the absolute acceleration, eliminating any inaccuracy arising from this approximation.

Interpretation of Numerical Values. The PFA value is best understood through its **amplification factor with respect to PGA**.

- **For very stiff components** ($T_n \rightarrow 0$): The component moves in unison with the ground. Therefore, the PFA Spectrum will tend towards the PGA value at zero period.
- **For components in resonance:** If the period of a non-structural component coincides with one of the fundamental periods of the main building, a “cascade resonance” phenomenon can occur, generating very high amplification factors.
- **Typical Values:** For design earthquakes, it is not uncommon to find that the peak of the PFA Spectrum is on the order of **2 to 4 times the PGA value**. For example, for an earthquake with a PGA of 0.5g, the PFA Spectrum could reach values of 1.0g to 2.0g or even higher in certain period ranges. This amplification is the reason why extensive damage to contents and non-structural elements is often observed even in buildings that have withstood the earthquake well structurally.

The program reports the maximum PFA value found across the entire range of periods. This information is of vital importance for the design and anchorage of non-structural components, the safety assessment of sensitive equipment, and the estimation of economic losses in a seismic event.

3.3.7 Note on the Interpretation of Intensity Measures

Throughout this section, various intensity measures have been presented, and for some of them, approximate correlation tables with the Modified Mercalli Intensity (MMI) Scale. It is essential that the reader interprets these tools with professional judgment and is aware of their nuances.

Each intensity measure is designed to capture a different aspect of an earthquake's damage potential: total energy (I_A), the threshold for damage initiation (CAV), the demand on a range of structures (HI), the asymmetry of the motion (I_D), or the response in non-structural components (PFA). Because they measure different physical phenomena, **it should not be expected that their respective correlations with MMI will produce an identical value** for the same ground motion.

The correlation tables presented in this chapter constitute a **non-standardized order-of-magnitude reference**, compiled by the author based on the literature, for the sole purpose of providing the engineer with a practical context for interpreting the numerical values. The choice of the most relevant intensity measure and the final damage assessment will always depend on the type of structure, the site conditions, and the specific objectives of the analysis being performed.

3.4 Analysis in the Frequency Domain

3.4.1 The Fourier Transform and the Fourier Amplitude Spectrum (FAS)

The **Fourier Transform** is the mathematical tool that decomposes a time signal, $a_g(t)$, into its constituent frequency components. The result is a complex function, $A(\omega)$, whose modulus, $|A(\omega)|$, is known as the **Fourier Amplitude Spectrum** (FAS). The FAS reveals the amplitude of each frequency harmonic present in the signal, but by discarding the phase information, it does not describe how these amplitudes are distributed over time.

3.4.2 The Power Spectral Density Function (PSDF)

While the Fourier Amplitude Spectrum (FAS) describes the frequency content of a particular deterministic signal, to model an accelerogram as a realization of a stochastic process, the conceptually correct tool is the **Power Spectral Density Function** (PSDF), denoted as $G(\omega)$. The PSDF does not describe a single signal but the statistical properties of an *ensemble* or family of signals. Its function is to define how the “power” of the signal is distributed in the frequency domain — which for a zero-mean signal like acceleration, is equivalent to its variance (σ^2).

The fundamental relationship is that the total area under the PSDF curve is the total variance of the signal:

$$\sigma^2 = \int_0^\infty G(\omega) d\omega$$

The **Wiener-Khinchin Theorem** establishes the formal relationship between the PSDF and the autocorrelation function of the process, solidifying it as one of the most important tools in random vibration analysis. For a comprehensive treatment of these concepts, reference texts, both classic [9] and modern with a computational focus [36], are recommended.

Physical Interpretation and Units. Unlike mechanical power (measured in watts), the “power” of the signal in this context refers to the contribution of each frequency to the total energy of the motion, or more precisely, to its mean square value. Therefore, if acceleration is measured in $[m/s^2]$, the units of the PSDF are $[(m/s^2)^2/Hz]$ or $[(m/s^2)^2/(rad/s)]$. This represents the variance (or “power”) per unit of frequency bandwidth. The integral of the PSDF over a specific frequency band, for example between 5 and 10 Hz, would give us the partial energy or variance of the accelerogram contained exclusively within that frequency range.

Relationship with Accelerogram Synthesis. The paramount importance of the PSDF for the **first phase** of this work is that it provides the direct bridge for the initial synthesis of accelerograms. As will be detailed in Chapter 6, an artificial accelerogram $a(t)$ is generated as a sum of a large number of sinusoids with random phases:

$$a(t) = \sum_{i=1}^N A_i \sin(\omega_i t + \phi_i)$$

The average power of a single sinusoid $A_i \sin(\omega_i t + \phi_i)$ is $A_i^2/2$. For the synthetic signal to be a realization of the stochastic process defined by the PSDF $G(\omega)$, the power of each discrete sinusoidal component must correspond to the power contained in the continuous frequency band $\Delta\omega$ that said sinusoid represents. This leads to the fundamental relationship:

$$\frac{A_i^2}{2} \approx G(\omega_i) \Delta\omega_i$$

From this, the required amplitude for each sinusoid can be derived:

$$A_i \approx \sqrt{2G(\omega_i) \Delta\omega_i}$$

This is why the PSDF, and not the FAS, is the necessary tool for simulation: it allows the generation of multiple accelerograms that, having different random phases, are temporally different, but being based on the same PSDF, are statistically equivalent in their energy and frequency content.

Fundamental Difference from the Response Spectrum. It is crucial not to confuse the PSDF (*Power Spectral Density Function*) with the response spectrum, as they answer different questions.

- The **PSDF** is an **intrinsic property of the ground motion**. It describes the energy distribution of the input signal itself (the accelerogram).

- The **Response Spectrum** describes the **effect of that motion** on a set of idealized structures. It measures the maximum response of an SDOF when subjected to that signal.

In summary, the PSDF characterizes the load, while the response spectrum characterizes the response to that load.

3.5 The Bridge Between Disciplines: Seismology and Engineering

The seismologist and the structural engineer analyze a seismic record for different purposes. The seismologist seeks to “invert” the recorded signal to deduce properties of the source and path. The engineer, in contrast, uses the record as an **external action** to predict the structural **response**. While the seismologist may be interested in the source spectrum, the engineer focuses on the response spectrum, which is the result of convolving the input signal with the oscillator’s transfer function. This difference in objectives is manifested in two key areas: the magnitude of accelerations and the meaning of the return period.

The “Problem” of Accelerations: Rock vs. Foundation. It is not uncommon for seismological models to predict peak ground accelerations (PGA) on rock that are significantly higher than those considered by engineers at the base of their structures. This is not an error but the consequence of analyzing the motion at two different points. The seismologist seeks to characterize the pure motion on bedrock (“outcrop motion”), which serves as the fundamental input signal for any site analysis. The engineer, however, needs to know the motion that effectively reaches the foundation, after the signal has been filtered by the superficial soil strata. For high-intensity earthquakes, these soil strata exhibit highly **nonlinear** behavior: their stiffness degrades, and their damping capacity increases drastically. This nonlinear damping dissipates a very significant portion of the earthquake’s high-frequency energy, acting as a natural “shock absorber” that, while it may lengthen the signal’s period, often reduces the PGA value that ultimately reaches the structure. Therefore, both accelerations are correct, but they describe phenomena at different locations.

Hazard vs. Risk: The Meaning of the Return Period. The terms “return period” and “probability of exceedance” are the language of seismic hazard, but their application differs between disciplines.

- **The Role of the Seismologist (Hazard Analysis):** The seismologist performs a **Probabilistic Seismic Hazard Analysis** (PSHA) to quantify the **hazard**. The result is a “hazard curve” that shows the annual probability of exceeding a certain level of ground motion. The return period is simply the inverse of this annual probability (e.g., an annual probability of $1/2475$ corresponds to a 2475-year return period). The seismologist provides the complete “menu” of possible hazard levels and their probability of occurrence.

- **The Role of the Engineer (Risk Management):** The engineer, guided by society through codes and regulations, manages **risk**, which is conceptually defined as:

$$\text{Risk} = \text{Hazard (Probability)} \times \text{Vulnerability (Consequence)}$$

The engineer does not use all return periods; they select one from the seismologist’s “menu” based on a socio-economic decision. For a conventional building, the code may require the use of the “Maximum Considered Earthquake” (MCE), which often has a return period of 2475 years (2

Does a 5000-year return period make sense? For an office building, probably not. The cost of designing for such a rare event would outweigh the benefit. However, for a **critical infrastructure** — a dam, a nuclear power plant, a major bridge that is a vital evacuation route — the **consequence** of failure is socially unacceptable. Although the probability of an earthquake with a 5000- or 10,000-year return period is extremely low, the cost of the catastrophe (in lives and economy) is so astronomically high that the resulting risk remains significant and must be managed. In these cases, it makes perfect sense to design for events much rarer than those considered for conventional buildings.

Tools like `earthquake_2025` are, therefore, the final bridge connecting these disciplines. The engineer selects a hazard level (a design spectrum for a given return period, based on a risk assessment), and `earthquake_2025` translates that abstract, probabilistic objective into a deterministic time history, ready to be used in the most advanced structural analyses required by modern engineering.

3.6 The Modern Definition of Hazard: Probabilistic Analysis (PSHA)

To connect all the preceding concepts and understand how the design spectra used in modern codes are generated, it is essential to introduce the method that underpins them: **Probabilistic Seismic Hazard Analysis** (PSHA) [18, 38]. PSHA is the mathematical framework that allows us to answer the fundamental question: *“At a given location, what is the probability that the ground motion will exceed a certain intensity level during a specific period of time?”*.

Philosophy of PSHA. Unlike a deterministic analysis that would focus solely on the “worst-case” earthquake from the nearest fault, PSHA rigorously integrates the uncertainties inherent in the seismic phenomenon. It considers all seismogenic sources that can affect a site, from distant faults capable of generating mega-earthquakes to local faults that produce smaller but more frequent earthquakes. For each of these sources, the probability of earthquakes of all possible magnitudes occurring is evaluated.

Ingredients of the Analysis. A PSHA combines three key components:

1. **Seismic Recurrence Models:** For each seismic source, a law is established that defines the frequency at which earthquakes of different magnitudes occur. This allows for quantifying the probability that an event of a given magnitude will be generated at that source.
2. **Ground Motion Prediction Equations (GMPEs):** The second crucial ingredient is the **GMPEs**. These are empirical relationships, developed from databases of thousands of real seismic records (e.g., [12]), that predict how wave energy attenuates with distance. For an earthquake of magnitude M at a distance R , a GMPE provides the median expected value for a ground motion parameter (like PGA or spectral acceleration $S_a(T)$), as well as the standard deviation (σ) that quantifies the variability and uncertainty of that prediction.
3. **Probabilistic Integration:** Finally, PSHA uses the total probability theorem to integrate the contributions from all scenarios (all possible magnitudes on all faults at all possible distances). The result of this complex integration is the **annual rate of exceedance** for a given motion level, i.e., the probability that said motion will be surpassed in one year.

Key Results of PSHA. The analysis produces two fundamental results for engineering:

- **Seismic Hazard Curve:** This is a graph that relates an intensity parameter (e.g., PGA) to its annual rate of exceedance. The return periods used in regulations are derived from this curve.
- **Uniform Hazard Spectrum (UHS):** By calculating a hazard curve for each spectral period, a response spectrum can be constructed in which all its ordinates (all its points) have the same probability of being exceeded. This UHS is the scientific and objective basis from which the smoothed design spectra presented in Appendix B are derived.

PSHA is, therefore, the engine that translates geology and seismology into the probabilistic language of hazard, providing the structural engineer with the rigorous starting point for earthquake-resistant design.

3.7 Physical Limitations in Spectrum Matching

One of the deepest conceptual difficulties in the simulation of artificial earthquakes arises when confronting the program's objective with the physical properties of structural response. The purpose of `earthquake_2025` is to generate a single acceleration time history whose response spectrum is compatible with a target spectrum. However, when this target spectrum contains a perfectly flat and horizontal acceleration plateau—as is the case with virtually all code-based design spectra—the objective becomes, in a strict sense, physically impossible to achieve.

Understanding this impossibility is crucial, as it reveals that the challenge is not algorithmic, but fundamental. This section mathematically demonstrates why no finite-duration accelerogram can generate a flat and horizontal response spectrum, thus explaining why the result of a matching program will always exhibit an unavoidable ripple in such zones.

Proof by Asymptotic Analysis. The proof is based on analyzing the physical behavior of a single-degree-of-freedom (SDOF) oscillator at the two extremes of the period spectrum: extremely long and extremely short periods.

First, let us consider the limit for very long periods ($T \rightarrow \infty$).

1. The fundamental relationship between the Pseudo-Acceleration Spectrum (PSA) and the Displacement Spectrum (SD) is:

$$PSA(T) = \left(\frac{2\pi}{T}\right)^2 \cdot SD(T)$$

2. An oscillator with a nearly infinite period is so flexible that its mass remains practically motionless in inertial space while the ground moves beneath it. Its relative displacement, $u(t)$, tends to be the opposite of the ground displacement, $d_g(t)$. Therefore, its maximum relative displacement, $SD(T)$, converges to the Peak Ground Displacement (PGD), which is a finite and positive value for any real earthquake.

$$\lim_{T \rightarrow \infty} SD(T) = \text{PGD}$$

3. Substituting this limit into the first equation yields:

$$\lim_{T \rightarrow \infty} PSA(T) = \lim_{T \rightarrow \infty} \left(\frac{2\pi}{T}\right)^2 \cdot \text{PGD} = 0$$

This demonstrates that any response spectrum of a real accelerogram **must tend to zero** for long periods. A horizontal, non-zero plateau violates this fundamental condition.

Analogously, we can analyze the limit for extremely short periods ($T \rightarrow 0$).

- An oscillator with a nearly zero period is infinitely rigid. As such, it experiences no relative displacement; it moves in unison with the ground.
- The absolute acceleration of its mass is, therefore, identical to the ground acceleration, $a_g(t)$, at every instant.
- The value of the Pseudo-Acceleration Spectrum, which represents the maximum absolute acceleration, must then converge to the Peak Ground Acceleration (PGA).

$$\lim_{T \rightarrow 0} PSA(T) = \max_t |a_g(t)| = \text{PGA}$$

This demonstrates that any response spectrum **must start at the PGA value** of the accelerogram that generates it. A horizontal plateau could only meet this condition if its constant value were exactly the PGA. In such a case, the accelerogram would also have a constant acceleration at all times, which is not consistent with an earthquake accelerogram.

Between these two asymptotic extremes lies the behavior of intermediate oscillators, which is the most relevant for structural engineering. Unlike the limit cases, the response of an intermediate oscillator is dominated by **dynamic amplification**. Any real accelerogram has a non-uniform frequency content; that is, its energy is concentrated in certain frequency bands and is scarce in others. An oscillator's response will be massively amplified if its natural period, T_e , coincides with one of these energetic bands (resonance), and it will be much smaller if it coincides with a low-energy zone. Consequently, the response spectrum of any physically plausible time signal *must* have peaks and valleys that reflect this resonant interaction. The notion of a perfectly flat plateau would imply that the accelerogram possesses a miraculously calibrated energy distribution to produce the same maximum response in all oscillators, regardless of their period, which contradicts the fundamental nature of the resonance phenomenon.

The foregoing analysis, therefore, conclusively demonstrates that the response spectrum of any given non-constant accelerogram cannot exhibit a horizontal plateau.

Consequences for the Interpretation of Results. This demonstration has a direct practical consequence for the interpretation of the results from `earthquake_2025`. The program attempts to generate a single time signal whose spectrum fits an idealized envelope that is, in part, physically unrealizable. As a result, the calculated spectrum will inevitably exhibit a slight “ripple” or undulations along the target plateau.

It is worth remembering that code-based spectra do not represent a single physical event, but a **statistical envelope** designed to cover the uncertainty of multiple possible scenarios. For this reason, they maintain flat plateaus or slowly decaying branches that no individual earthquake possesses on its own. The “ripple” is, therefore, the mathematical manifestation of the tension between the physics of a single signal and the conservatism of a design envelope.

These undulations should not be interpreted as a flaw in the algorithm, but as the **signature of a physically plausible solution**. A perfectly flat and horizontal spectrum would imply an infinite or non-physical time signal. The objective of “super-compatibility” is, therefore, not to eliminate these undulations—which is impossible—but to minimize their amplitude and ensure that the calculated spectrum oscillates as closely as possible around the target line, guaranteeing that the energy in that frequency band is, on average, correct.

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Chapter 4

From Seismic Action to the Philosophy of Earthquake-Resistant Design

The connection between the characterization of a seismic motion and its practical application in the design of safe structures is the core of earthquake engineering. This chapter addresses the fundamental concepts that enable this transition, from the essential tool of the elastic response spectrum to the principles of performance-based design, topics that have been treated in depth by classic texts such as that of Okamoto [46].

4.1 The Elastic Response Spectrum: The Fundamental Tool

The analysis of structural response to an arbitrary seismic excitation $a_g(t)$ is conceptually simplified by studying a Single-Degree-of-Freedom (SDOF) System, whose fundamental theory is established in classic texts on mechanical vibrations [21]. The equation of motion governing its relative response, $u(t)$, is:

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = -ma_g(t) \quad (4.1)$$

Dividing by the mass m and using the definitions of the undamped natural frequency, $\omega_n = \sqrt{k/m}$, and the critical damping ratio, $\xi = c/(2m\omega_n)$, the equation is rewritten in its canonical form:

$$\ddot{u}(t) + 2\xi\omega_n\dot{u}(t) + \omega_n^2u(t) = -a_g(t) \quad (4.2)$$

The **response spectrum** is the graphical representation of the maximum value of a response quantity of interest (displacement, velocity, or acceleration) experienced by the SDOF, as a function of its natural period $T_n = 2\pi/\omega_n$, for a fixed damping ratio ξ . This concept, decisively advanced by the pioneering work of Housner [33], is the solution to the “forward problem”: given a ground motion, *what is the maximum demand it will impose on a family of possible idealized structures?* [15].

4.1.1 Construction, Interpretation, and the “Pseudo” Nature of Spectra

For a given accelerogram, the time-history response $u(t)$ of Equation 4.2 is obtained by solving the classic **Duhamel integral**. Since this integral rarely has an analytical solution for an arbitrary function $a_g(t)$, it must be solved numerically. The `earthquake_2025` program has implemented a high-precision state-space solution that can be divided into multiple time sub-steps to ensure the convergence and accuracy of the result.

Once the response history $u(t)$ is obtained for each oscillator of period T_n , its maximum relative displacement, $SD(T_n, \xi) = \max_t |u(t)|$, is determined and recorded. From this fundamental value of each oscillator, the other two spectral ordinates of interest (velocity and acceleration) are approximately calculated. It is in this approximation that the distinction between “real” and “pseudo” values arises, as explained below:

Pseudo-Velocity and Pseudo-Acceleration. For mathematical convenience and to facilitate graphical representation, the velocity and acceleration ordinates for each oscillator of period T_n are not calculated as the true maximums of the histories $\dot{u}(t)$ and $\ddot{u}_{total}(t)$, but are instead approximated directly from the spectral displacement using the expressions $PSV(T_n, \xi) = \omega_n \cdot SD(T_n, \xi)$ and $PSA(T_n, \xi) = \omega_n^2 \cdot SD(T_n, \xi)$.

- **Pseudo-Velocity Spectrum (PSV):** This value is of great physical importance, as the maximum elastic strain energy stored in the oscillator is $E_s = \frac{1}{2}k \cdot SD^2 = \frac{1}{2}m \cdot PSV^2$. The PSV is an excellent approximation of the true maximum relative velocity $\dot{u}(t)$ for systems with low damping ($\xi \leq 5\%$), but the error can increase for higher damping levels.
- **Pseudo-Acceleration Spectrum (PSA):** This value represents exactly the maximum internal elastic force per unit mass ($f_s/m = k \cdot SD/m = \omega_n^2 \cdot SD$). It is important to note that the PSA is **not** an approximation of the maximum *relative* acceleration, but of the maximum *absolute* acceleration, $\max_t |\ddot{u}(t) + a_g(t)|$. Again, the approximation is very good for low damping but can diverge for high damping.

The Utility and Accuracy of “Pseudo” Quantities. The fundamental reason for adopting this convention of using “pseudo” quantities is their enormous practical utility. For example, the mathematical interdependence between the three ordinates allows for the creation of the **tripartite plot on a logarithmic scale** (see Subsection 4.1.2), a visualization and design tool of incalculable value.

The magnitude of the error introduced by this approximation depends fundamentally on the damping level ξ and the frequency content of the signal [15]. For the damping levels representative of civil engineering structures — typically between 2% and 3% for steel structures and 5% for reinforced concrete structures — the error is negligible for most practical applications. Studies have shown that for $\xi \leq 5\%$, the difference between “pseudo” and real quantities is usually less than 4%, with the largest discrepancies concentrated in the very high-frequency range (very short periods), a zone that does not typically govern the design of conventional buildings [16].

Recommendation for Time-History Analysis. In the context of time-history analyses, it is common practice in many finite element programs to apply the seismic excitation as a prescribed displacement history at the base nodes, rather than an acceleration history. The author strongly recommends this approach for two fundamental reasons:

- First, it is conceptually and numerically more accurate, as finite element solution algorithms operate internally with displacement degrees of freedom. If accelerations are incorporated, the program has to integrate them twice to calculate displacements, a process where numerical errors can arise.
- Second, this approach completely nullifies any discussion about the “small error” introduced by “pseudo” quantities. By applying the complete displacement history, the structural analysis calculates the true response of the system, and the distinction between “pseudo” and real quantities ceases to be relevant, as the calculation is based on the full kinematics and not on values that have a “small” level of approximation “error”.

4.1.2 The Tripartite Representation on a Log-Log Scale

The simultaneous plotting of the three spectra (SD, PSV, PSA) on a single graph with logarithmic scales on both axes [44] reveals a powerful interpretation tool. Due to the mathematical relationships between them, the spectra tend to form straight lines in different regions of the graph, defining three control zones:

1. **Short-Period Zone (acceleration-controlled):** For very stiff structures ($T_n \rightarrow 0$), the relative displacement is minimal, and the mass moves in unison with the ground. The absolute acceleration of the mass tends to the peak ground acceleration, $PSA(T_n \rightarrow 0) \rightarrow PGA$.
2. **Intermediate-Period Zone (velocity-controlled):** In this region, the PSV spectrum tends to amplify the peak ground velocity. This is the zone where the maximum energy demand is usually observed.
3. **Long-Period Zone (displacement-controlled):** For very flexible structures ($T_n \rightarrow \infty$), the mass tends to remain static in space while the ground displaces beneath it. Therefore, the maximum relative displacement tends to the peak ground displacement, $SD(T_n \rightarrow \infty) \rightarrow PGD$.

Graphical Interpretation. Figure 4.1 shows an example of an idealized spectrum, plotted on this special logarithmic paper. In this representation, the three control zones are clearly manifested. The short-period region (acceleration-controlled) tends to an ascending straight line, the intermediate-period region (velocity-controlled) becomes a horizontal line, and the long-period region (displacement-controlled) tends to a descending straight line. The diagonal grids allow for direct reading of the Pseudo-Acceleration (dashed lines) and Spectral Displacement (dotted lines) values. The numerical data used to generate this figure are presented in Table 4.1.

A representation of a spectrum from a real example with 10% damping has also been placed on the graph.

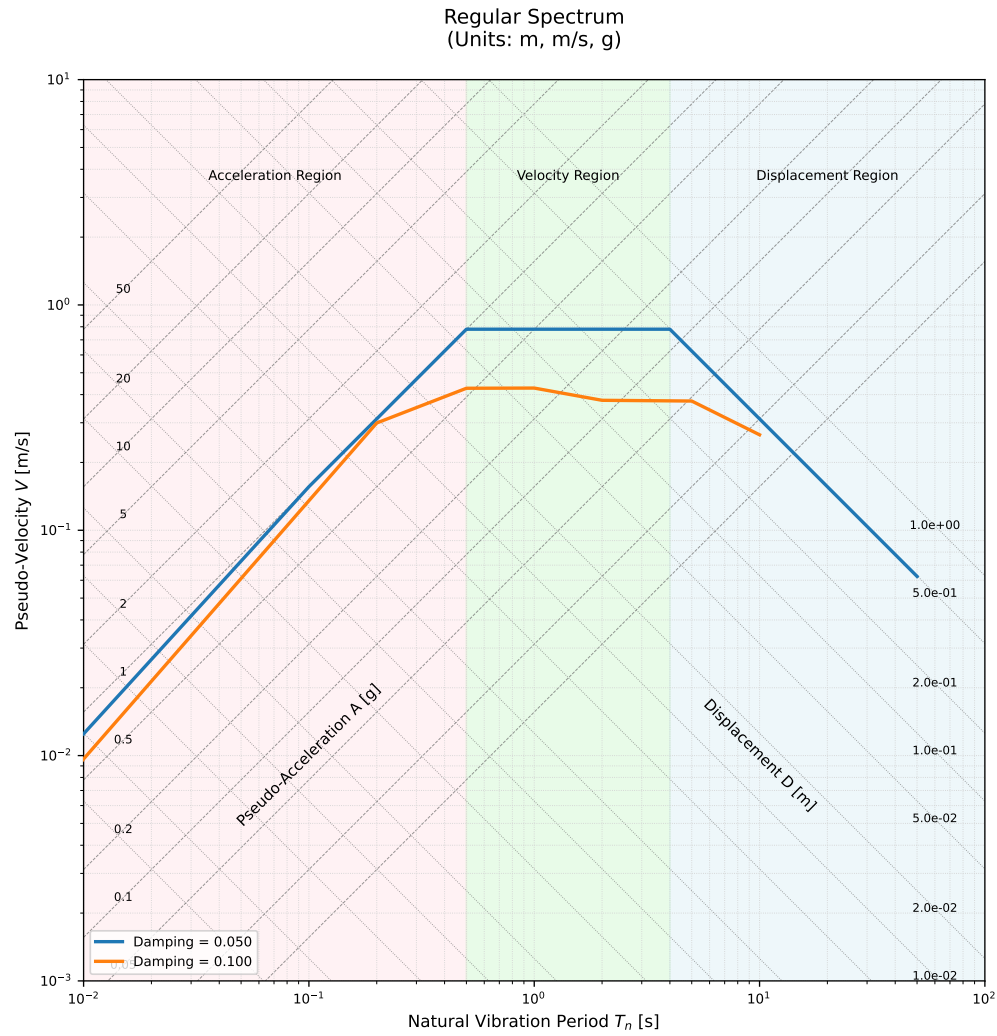


Figure 4.1: Example of an idealized response spectrum in a tripartite representation format. The shaded areas indicate the regions controlled by acceleration, velocity, and displacement, respectively.

Period T (s)	PSA (g)	PSV (m/s)	SD (m)
0.0100	0.8000	0.0125	0.000020
0.1000	1.0000	0.1561	0.002485
0.5000	1.0000	0.7804	0.062101
4.0000	0.1250	0.7804	0.496810
10.0000	0.0200	0.3121	0.496810
50.0000	0.0008	0.0624	0.496810

Table 4.1: Points of the idealized response spectrum for 5% damping.

4.1.3 The Elastic Spectrum as a Reference Demand

At this point, it is vitally important to pause and clarify the profound meaning of the elastic spectrum and its role in engineering, contrasting it with the concept of an inelastic spectrum.

An **elastic response spectrum**, as described, answers a purely theoretical question: if a structure were to behave like a perfect spring, with infinite strength and without suffering any damage, *what would be the maximum force it would experience during the earthquake?* It is, therefore, a measure of the **force demand** on an idealized system that cannot yield. Constructing a real building capable of elastically resisting these forces for a severe earthquake would, in most cases, be economically and technically unfeasible.

This is where the concept of an **inelastic response spectrum** comes in. It is constructed from a model that does have a limited strength and, once reached, yields and dissipates energy through hysteresis loops. This spectrum does not answer the question of maximum force, but a question much more relevant to real design: for a structure with a given deformation capacity (ductility), *what is the strength it needs to survive the earthquake without collapsing?* It is, therefore, a measure of the **deformation (or ductility) demand**.

So, *why is the elastic spectrum, and not the inelastic one, used as the target for generating accelerograms with the earthquake_2025 program?* The answer is the key to modern analysis methodology:

- The purpose is to generate a time history that contains the **total and unmitigated energy demand** of the seismic event, as defined by the code. The elastic spectrum is precisely that reference.
- This accelerogram is then used as input for a **Non-Linear Time History Analysis (NLTHA)** in a finite element software (such as Abaqus, as shown in Appendix C).
- It is the **nonlinear model of the structure** within that software that is responsible for simulating yielding, energy dissipation, and force redistribution. By subjecting the nonlinear model to the load from the accelerogram (compatible with the elastic spectrum), the analysis tells us what the true ductility demand and the level of damage to the structure are.

Similarly, the target elastic spectrum used to generate the artificial earthquake must correspond to the **damping level specified by the code**, which is typically 5%. If the structure to be analyzed has a different damping level (e.g., due to the addition of energy dissipators), this higher damping must be modeled in the structural analysis program, not in the definition of the input earthquake.

In summary, the process is separated into two clear responsibilities: `earthquake_2025` is responsible for creating a load signal that faithfully represents the code's elastic demand. The structural analysis program is responsible for calculating how a realistic, nonlinear structure responds to that load. Attempting to generate an accelerogram from an inelastic spectrum would be conceptually incorrect, as one would be "pre-processing" the nonlinear response before the structural analysis itself has even begun.

4.2 Nonlinear Behavior: The Reality Under Severe Earthquakes

4.2.1 Incursion into the Inelastic Range

The premise of purely elastic behavior is a valid idealization only for low-intensity earthquakes. To economically resist the action of a severe earthquake, modern earthquake-resistant design explicitly relies on the structure's ability to enter the **nonlinear** range. This means that the relationship between internal forces and displacements ceases to be proportional. Materials, such as reinforcing steel or structural steel, reach their yield point and begin to deform plastically.

4.2.2 Ductility and Hysteretic Energy Dissipation

A structure's ability to survive a severe earthquake lies not in its raw strength, but in its **ductility**. Ductility is defined as the ability of a material, section, element, or structural system to sustain large inelastic deformations without a significant degradation of its strength. Quantitatively, the ductility factor, μ , is defined as the ratio of the ultimate displacement, δ_u , to the yield displacement, δ_y .

When a structure enters the inelastic range under a cyclic load like that of an earthquake, its force-displacement behavior describes **hysteresis** loops. The area enclosed by each of these loops on the force-displacement graph represents the energy that is dissipated by the structure, primarily in the form of heat and localized damage (e.g., steel yielding, concrete cracking). This mechanism of **hysteretic energy dissipation** is the most important form of damping in a severe earthquake and is what allows the structure to manage the enormous amount of energy imparted to it by the earthquake without collapsing.

4.2.3 The Insufficiency of Modal Response Spectrum Analysis

Modal response spectrum analysis, the most common design method, is fundamentally based on the principle of superposition. This principle states that the total response of a linear system to a set of loads is the sum of the responses to each load applied individually. However, when a structure becomes nonlinear, its stiffness is no longer a constant: it changes at every instant in time as a function of its previous deformation history. Since stiffness (and resisting forces) is not a linear function of displacement, the system is no longer linear, and the principle of superposition loses all validity.

Consequently, modal response spectrum analysis is incapable of predicting the actual sequence of yielding, the distribution of damage, the magnitude of residual deformations, or the time history of the response. The only rigorous analytical tool for evaluating the behavior of a nonlinear structure is the direct, step-by-step numerical integration of the nonlinear equation of motion, a method known as **Non-Linear Time History Analysis (NLTHA)**.

4.3 Principles and Strategies of Earthquake-Resistant Design

Since nonlinear behavior is expected, earthquake-resistant design does not focus on preventing damage, but on controlling it. The structure is designed so that inelastic damage occurs in predictable locations and in a controlled manner, forming a stable collapse mechanism.

4.3.1 The Hierarchy of Strength: “Strong Column - Weak Beam”

The “Strong Column - Weak Beam” (SCWB) philosophy is perhaps the most important principle of capacity design for reinforced concrete and steel moment frames. Its goal is to dictate and control the inelastic energy dissipation mechanism in advance by predetermining which elements should yield and which should remain fundamentally elastic to ensure the overall stability of the structure.

The principle consists of designing the columns to be significantly stronger than the beams that meet at a joint. In this way, the beams are designed to act as structural “fuses”: they are the elements that will be sacrificially yielded in a controlled manner. By forcing the yielding (the formation of “plastic hinges”) to occur at the ends of the beams, the structure develops a ductile and stable global collapse mechanism. The inelastic deformation is distributed among the ends of numerous beams throughout the height of the building, allowing the structure as a whole to dissipate an enormous amount of energy while maintaining its integrity and its ability to support gravity loads.

Conversely, if the columns are weaker and yield before the beams, there is a risk of forming a “soft story” or “weak story”. In this catastrophic scenario, the entire inelastic deformation demand is concentrated in the columns of a single level. This leads to a rapid degradation of that story’s strength and stiffness, generating large lateral displacements which, acting together with gravity loads, are amplified by second-order (P-Delta) effects. This phenomenon creates an instability that can lead to a brittle and progressive collapse of the entire building.

To ensure this ductile behavior and avoid the soft-story mechanism, modern earthquake-resistant codes (such as ASCE 7 or Eurocode 8) impose explicit capacity design requirements, demanding that the sum of the flexural strengths of the columns meeting at a joint be significantly greater than that of the beams framing into it in the same direction [48].

4.3.2 The Importance of Structural Configuration

Experience has shown that structures with simple and regular geometric configurations in plan and elevation have far superior seismic performance. Irregularities should be avoided, such as:

- **Torsional Irregularities:** Large eccentricities between the center of mass and the center of rigidity.
- **Stiffness or Strength Irregularities:** Such as the presence of a “soft story” or “weak story”.

- **Discontinuities in the Resisting System:** Such as the interruption of columns or walls on lower floors.

High structural **redundancy**, with multiple load paths, is also a highly desirable property, as it allows for the redistribution of forces if an element fails.

4.3.3 Ductile Detailing

Ductility, assumed in the design through a response modification factor (R or q), is a theoretical property that can only be realized if the structural elements and their connections are capable of sustaining large inelastic deformations and multiple load cycles without significant degradation of their strength. The design of construction details to ensure this behavior is known as **ductile detailing**. Neglecting this aspect is one of the most common causes of structural failure in earthquakes, even in buildings that, on paper, had adequate strength.

In Reinforced Concrete Structures. In concrete, ductile detailing focuses on the zones of potential plastic hinge formation (beam ends, and column base and top). The key is to provide adequate **confinement** of the concrete core. This is achieved through transverse reinforcement consisting of closely spaced stirrups (or hoops in circular columns) with 135-degree seismic hooks. This transverse reinforcement serves a critical triple function:

1. **Prevents Shear Failure:** It provides the necessary shear strength for the section to develop its full flexural capacity, ensuring a ductile flexural failure rather than a brittle shear failure.
2. **Prevents Buckling of Longitudinal Bars:** When subjected to compression during load cycles, longitudinal bars tend to buckle. The closed stirrups provide the necessary lateral bracing to prevent this.
3. **Increases the Deformation Capacity of Concrete:** The confinement provided by the transverse steel drastically increases the compressive strength and, above all, the ultimate strain of the core concrete, allowing the plastic hinge to sustain large curvatures without disintegrating.

In Steel Structures. In steel, which is an intrinsically ductile material, detailing focuses on the **connections**. The goal is to ensure that yielding occurs in the beam and not in the connection elements (bolts, welds), following a local strength hierarchy. A well-detailed connection must be able to withstand the large inelastic rotations of the plastic hinge that forms in the adjacent beam without suffering a brittle fracture or degradation from low-cycle fatigue. The design must avoid stress concentrations and ensure that all connection components (plates, stiffeners, etc.) have adequate strength and ductility [48].

4.4 Performance-Based Design (PBD)

Performance-Based Design (PBD) is an evolution of the earthquake-resistant design philosophy that seeks to explicitly quantify and verify the behavior of a structure against different levels of seismic hazard. The approach is structured around three key components:

- **Performance Levels:** Acceptable damage states for the structure are defined. The most common are:
 - **Immediate Occupancy (IO):** Minimal damage, mainly non-structural. The structure is safe to be occupied immediately after the earthquake.
 - **Life Safety (LS):** Significant and repairable structural damage, but collapse is prevented, and the safe evacuation of occupants is ensured.
 - **Collapse Prevention (CP):** Severe structural damage, close to collapse, but structural integrity is maintained to allow for evacuation and prevent loss of life.
- **Hazard Levels:** Earthquakes with different probabilities of occurrence are defined, derived from Probabilistic Seismic Hazard Analysis (PSHA):
 - **Service Level Earthquake (SLE):** A frequent seismic event, with a high probability of occurrence during the structure's service life (e.g., 50% in 50 years).
 - **Design Basis Earthquake (DBE):** The reference hazard level in most codes, with a low probability of exceedance (typically 10% in 50 years, equivalent to a 475-year return period).
 - **Maximum Considered Earthquake (MCE):** The most severe level of hazard, with a very low probability of exceedance (typically 2% in 50 years, equivalent to a 2475-year return period), which the structure must be able to resist without collapsing.
- **Performance Objectives Matrix and Verification Process:** The PBD philosophy is formalized in a **performance objectives matrix**, which defines the acceptable damage level (Performance Level) for each considered seismic hazard level. For example, a typical matrix for an office building might require:
 - Under the frequent earthquake (SLE) hazard, the structure must remain at the Immediate Occupancy (IO) level.
 - Under the design basis earthquake (DBE) hazard, the structure must meet the Life Safety (LS) level.
 - Under the maximum considered earthquake (MCE) hazard, the structure must ensure Collapse Prevention (CP).

The core of PBD is the **quantitative verification** that the structural design meets these objectives. This verification process is performed using a Non-Linear Time-History Analysis (NLTHA). The analysis outputs the complete time history of the structure's response, from which the **Engineering Demand Parameters (EDPs)** are extracted. EDPs are the metrics that quantify physical damage to the structure. The most important are:

- **Maximum Interstory Drifts:** The maximum relative displacement between consecutive floors, divided by the story height. It is the primary indicator of damage to non-structural elements and of the potential for instability due to P-Delta effects.
- **Maximum Plastic Rotations:** The maximum inelastic rotation in the plastic hinges of beams and columns. It is the primary indicator of localized damage to structural elements.
- **Maximum Floor Accelerations:** The maximum acceleration experienced at each level. It is the primary indicator of the demand on non-structural components and contents.

Finally, these calculated EDPs are compared against the **acceptance limits** (damage threshold values) that define each Performance Level. These limits are tabulated in specialized codes like ASCE 41 for the rehabilitation of existing buildings [5], or are derived from loss assessment methodologies like FEMA P-58 [25]. If the calculated EDPs are less than the acceptance limits for each hazard-performance pair, the design is considered to have been successfully verified.

4.5 The Interaction between the Earthquake and the Structure

4.5.1 Resonance and the Influence of Frequency Content

The phenomenon of **resonance** occurs when an excitation frequency is close to one of the structure's natural frequencies, causing a massive dynamic amplification of the response. In the seismic context, this means that the hazard is greatest when the **fundamental period of the structure** (T_e) is close to the **dominant period of the seismic excitation**.

It is crucial to understand that the dominant period of the excitation reaching the base of the structure is not necessarily that of the seismic source, but is strongly influenced by local soil conditions. A soft soil deposit, for example, acts as a dynamic filter that amplifies seismic waves with periods close to its own **fundamental site period** (T_s). Therefore, in practice, the most dangerous resonance condition occurs when the structure's period coincides with the site's period ($T_e \approx T_s$), as the soil will amplify precisely the frequencies that are most damaging to the structure. This interaction defines the vulnerability of different structural typologies to different types of earthquakes:

- **High-frequency earthquakes (short excitation periods):** Typical of moderate-magnitude, near-field events on rock. They are especially dangerous for low-rise, stiff structures (masonry buildings, short-span bridges), which have short periods.
- **Low-frequency earthquakes (long excitation periods):** Characteristic of large-magnitude, far-field earthquakes or sites on deep, soft soils. They are very demanding for tall, flexible structures (skyscrapers, cable-stayed bridges), which have long fundamental periods.

Good earthquake-resistant design seeks to avoid this coincidence of periods, for example, by not constructing tall, flexible buildings in soft soil zones.

4.6 Seismic Action in Design Codes

The **design spectrum** found in codes is not the response spectrum of a single earthquake, but a smoothed **probabilistic envelope**. It is constructed from seismic hazard studies that consider all possible earthquakes that can affect a site, to define an action with a certain probability of being exceeded in a given time frame (e.g., 10% in 50 years for the DBE).

Codes have evolved. Older, **force-based** approaches started with the elastic spectrum and reduced it by a response modification factor (R) to obtain design forces. This R factor attempted to implicitly account for the system's ductility and overstrength. Modern, **displacement-based** approaches recognize that damage is better correlated with displacements than with forces, and they focus on verifying that the deformation demands calculated through more sophisticated analyses (like PBD) are less than the deformation capacities of the structural elements [49].

4.7 Corollary: The Role of the Elastic Spectrum in Advanced Analysis

This chapter has established a conceptual bridge between seismic action, represented by the response spectrum, and the philosophy of modern structural design. The journey has taken us from the definition of the elastic spectrum as a measure of force demand to the understanding that the survival of structures under severe earthquakes depends on their ability to enter the nonlinear range in a ductile and controlled manner.

It has been shown that, while the elastic spectrum is a theoretical tool that assumes infinite strength, it is precisely this characteristic that makes it the indispensable starting point for advanced analysis. It defines the total, unmitigated energy demand of the seismic hazard.

This is where the `earthquake_2025` program plays its fundamental role. The program's task is not to simulate the nonlinear response of a structure — that is the responsibility of finite element analysis software — but to generate a ground motion time history that is perfectly compatible with the demand defined by the code's elastic response spectrum.

By providing a history of accelerations, velocities, and displacements that encapsulates all the frequency, amplitude, and duration information implicit in the spectrum, `earthquake_2025` generates the realistic and rigorous excitation that serves as input for non-linear time-history analyses (NLTHA). These analyses, as has been shown, are the only tool capable of reliably verifying the performance objectives (PBD), ductile detailing, and seismic protection strategies that define state-of-the-art earthquake-resistant engineering.

In essence, the elastic response spectrum is the objective; `earthquake_2025` is the translator; and the nonlinear analysis is the verification.

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Part II: Methodology, Application, and Analysis Context

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Chapter 5

User Guide and Analysis of Results

5.1 Introduction and Tool Context

This chapter constitutes the practical guide of the monograph, focusing on the use of the `earthquake_2025` program. This software is the computational materialization of the methodology for generating artificial accelerograms compatible with a response spectrum, the development of which began in the author's doctoral thesis, titled “**Una Metodología Para el Análisis Numérico del Comportamiento Resistente No Lineal de Presas de Hormigón Bajo Cargas Estáticas y Dinámicas**” (A Methodology for the Numerical Analysis of the Nonlinear Resistant Behavior of Concrete Dams Under Static and Dynamic Loads) [26], and was later formalized and published in Chapter 4 of the monograph “**IS-10 - Definición de la acción sísmica**” (IS-10 - Definition of the seismic action) [6].

The program, in its **2025 version**, represents not only a refinement of previous methods but also a significant advancement by incorporating a **two-phase hybrid methodology**. The following sections will detail the precise format of the input file required to run a simulation and, crucially, will provide a comprehensive guide for interpreting the numerical and graphical results that the program and its associated tools generate. The objective is to empower the user not only to run the program but also to critically understand the validity and meaning of each piece of data produced by this new and powerful algorithm.

5.2 The Input File: Format and Parameters

The `earthquake_2025` program reads all simulation parameters from a single text file. A detailed description of each parameter can be found in **Appendix A** of this monograph. As an example, Figure 5.1 shows a complete and valid input file, which will be used to generate the results analyzed in this chapter.

Four of the most important parameters in the input file, related to period and frequency, are `TSpecMin`, `TSpecMax`, `TConMin`, and `TConMax`. It is crucial to understand their distinct functions to configure the simulation correctly. These parameters define two different but related concepts: the *spectrum adjustment range* and the *signal content range*.

```

SIMULATION OF AN EARTHQUAKE FOR THE MONOGRAPH
|DISTANCE UNIT|
  [m]
| TSpecMin | TSpecMax | TContMin | TContMax | RFreqInc |
  0.01      10.0      0.0        0.0        0.004
|ENV_TYPE|  T_RISE  | T_PLATO  | DURATION  |
  3         0.0     0.0        20.0
|  POW  |  A0  |  ALPHA  |  BETA  |
  0.0     2.0     1.0     0.2
| DELTA_T |  PGA  |
  0.004   4.331
| G_ACCEL |
  9.81
| ISEED  | NDAMP  | NITERA  | NMOTION  | NPERIOD  | NPTS-SPECT |
  34641   2      30      10      250      7
| DAMP1  | DAMP2  | DAMP3  |
  0.05    0.40
| MaxStates | IMPROVE_THRS |
  5        0.01
|ICODE_SPECT_TYPE (0=SA_USER, 1=SV_USER)|
  0
|  T  |  SA  |
  0.001  4.335
  0.200  9.408
  0.500  5.366
  1.000  2.688
  2.000  1.187
  5.000  0.471
 10.000  0.167

```

Figure 5.1: Example of a complete input file for earthquake_2025.

5.2.1 Clarification on Period and Frequency Parameters

Spectrum Adjustment Range (TSpecMin and TSpecMax). These two parameters tell the program in which period range it should strive to make the response spectrum of the generated accelerogram match the target spectrum.

- **TSpecMin:** This is the shortest period (the highest frequency) from which the adjustment algorithm will begin to operate.
- **TSpecMax:** This is the longest period (the lowest frequency) up to which the adjustment algorithm will operate.

The program will take the target spectrum points provided by the user, interpolate them to a series of logarithmically spaced calculation points between **TSpecMin** and **TSpecMax**, and focus its effort on minimizing the error within this interval. Any part of the user-defined target spectrum that falls outside this range will be ignored by the adjustment process. Therefore, the user must ensure that this range covers the portion of the spectrum that is relevant to their structural analysis.

Signal Content Range (TContMin and TContMax). These two parameters, on the other hand, define the “bandwidth” of the accelerogram itself. They determine the range of frequencies (or periods) that will be physically present in the generated synthetic signal. The subroutine that synthesizes the accelerogram by summing sinusoids (see Equation 6.1) will only use sinusoidal components whose frequencies correspond to the period interval between **TContMin** and **TContMax**.

- **TContMin:** Corresponds to the highest frequency the signal will have
 $(\omega_{max} = 2\pi/T_{ContMin})$.
- **TContMax:** Corresponds to the lowest frequency the signal will have
 $(\omega_{min} = 2\pi/T_{ContMax})$.

Interaction and Usage Recommendations. In most practical cases, the content range should be equal to or slightly wider than the adjustment range. That is:

$$T_{ContMin} \leq T_{SpecMin} \quad \text{and} \quad T_{ContMax} \geq T_{SpecMax}$$

This configuration ensures that the generated signal contains all the necessary frequencies (and a small additional margin) to be able to adjust the spectrum in the range of interest. If, for example, one were to try to adjust a spectrum up to a period of 5 seconds (**TSpecMax**=5.0) but only include frequencies up to a period of 3 seconds in the signal (**TContMax**=3.0), the adjustment would be physically impossible, as the signal would lack the necessary low-frequency energy to excite the long-period oscillators. For this reason, if the user specifies a value of 0.0 for **TContMin** or **TContMax**, the program, for safety, adopts the values of **TSpecMin** and **TSpecMax** respectively, ensuring consistency between the two ranges.

5.2.2 PGA Consistency and High-Frequency Adjustment

A particular case of poor convergence, especially visible in the high-frequency (short-period) zone of the spectrum, deserves a detailed technical explanation. Often, a user may observe that while the fit is good over most of the spectrum, there is a persistent discrepancy near the shortest defined period. The root cause of this behavior lies in the consistency between the target spectrum and the specified PGA value.

By physical definition, the Peak Ground Acceleration (PGA) corresponds to the ordinate of the pseudo-acceleration (PSA) spectrum at a zero period, $PGA = PSA(T = 0)$. However, for numerical reasons (to avoid division by zero), the `earthquake_2025` program does not

allow the user to define a spectrum point at $T = 0$. Instead, the target spectrum is defined starting from a small but non-zero initial period (e.g., $T = 0.01$ s), and the PGA value is provided as an independent scaling parameter, PGA.

A potential conflict arises here. If the PGA value specified by the user is not consistent with the natural extrapolation of the target spectrum towards $T = 0$, the program faces two contradictory instructions:

1. The spectral adjustment algorithm will try to modify the Power Spectral Density Function (PSDF) so that the calculated spectrum matches the points defined by the user, including the point at the shortest period (e.g., $T = 0.01$ s).
2. The scaling subroutine will modify the entire accelerogram so that its maximum peak exactly matches the specified PGA.

If, for example, the code-based spectrum has a constant acceleration plateau of 0.5g in the short-period zone, but the user specifies a PGA of 1.0g, the program has no way to satisfy both objectives simultaneously. It will be forced to create a very abrupt discontinuity in the signal's actual spectrum between $T = 0$ (where the PSA will be 1.0g) and the first defined point (where it will try to make it 0.5g). This discontinuity is what manifests as a poor fit in that region. Therefore, a noticeable and persistent discrepancy in the high-frequency zone is almost certainly an indication that the specified PGA value is not consistent with the provided target spectrum. The correct practice is to ensure that the specified PGA matches the spectrum's acceleration ordinate at $T=0$, which for code-based spectra is usually the value of the constant acceleration plateau.

5.3 Anatomy and Interpretation of the Output File

The output file is a complete computational log. Its sequential structure allows for a detailed review of the entire process.

5.3.1 Echo of Parameters and PSDF Properties

The first section of the file presents an echo of the input data and the calculated properties of the initial Power Spectral Density Function (PSDF). This is crucial for data verification and reproducibility, as shown in Figure 5.2.

```

TARGET SPECTRUM AND FREQUENCY CONTENT DEFINITION:
  TARGET SPECTRUM - SMALLEST PERIOD ..... 0.0100 [s]
  TARGET SPECTRUM - LARGEST PERIOD ..... 10.0000 [s]
  MOTION CONTENT - SMALLEST PERIOD ..... 0.0100 [s]
  MOTION CONTENT - LARGEST PERIOD ..... 10.0000 [s]
  FREQUENCY GENERATION LN INCREMENT ..... 0.0040 [-]

TIME HISTORY PARAMETERS:
  TIME INCREMENT FOR SIGNAL ..... 0.0040 [s]
  TOTAL DURATION OF SIGNAL ..... 20.0000 [s]
  NUMBER OF POINTS IN SIGNAL ..... 5001 [#]

ANALYSIS TARGETS AND RESOLUTION:
  TARGET PEAK GROUND ACCELERATION (PGA) ..... 4.3310 [m/s2]
  GRAVITATIONAL ACCELERATION - FOR ARIAS_I ... 9.8100 [m/s2]
  NO. OF PERIODS FOR RESPONSE SPECTRUM ..... 250 [#]

SIMULATION LOOP PARAMETERS:
  RANDOM NUMBER SEED (INITIAL) ..... 34641 [#]
  NO. OF DIFFERENT MOTIONS TO GENERATE ..... 10 [#]
  NO. OF ITERATIONS FOR SPECTRUM MATCHING .... 30 [#]

TIME-DOMAIN ADJUSTMENT PARAMETERS (PHASE 2):
  MAXIMUM OPTIMIZATION STATES TO RUN ..... 5 [0-5]
  TIME-DOMAIN IMPROVEMENT THRESHOLD ..... 0.0100 [%]

DAMPING RATIOS FOR SPECTRUM CALCULATION:
  NO. OF DIFFERENT DAMPING RATIOS ..... 2 [#]
  DAMP1=0.05 DAMP2=0.40

ENVELOPE FUNCTION DEFINITION:
  ENVELOPE FUNCTION TYPE (EXPONENTIAL) ..... 3 [#]
  AMPLITUDE FACTOR (A0) ..... 2.0000 [-]
  RISE EXPONENT (ALPHA) ..... 1.0000 [-]
  DECAY EXPONENT (BETA) ..... 0.2000 [-]
  CALCULATED TIME TO PEAK ..... 2.0118 [s]
  CALCULATED PEAK VALUE ..... 1.0700 [-]

TARGET SPECTRUM:
  TARGET SPECTRUM TYPE (ICODE) ..... 0 (ACCELERATION)

NOTE: THE USER-SUPPLIED SPECTRUM HAS BEEN DENSIFIED
      FOR A PRECISE FIT TO ITS PIECEWISE-LINEAR SEGMENTS.

      NO. OF POINTS DEFINING TARGET SPECTRUM ..... 201 [#]

#   PERIOD      FREQUENCY    DISPL.SPECT.    VEL.SPECT.    ACCEL.SPECT.
   [s]         [Hz]         [m]           [m/s]        [m/s2]
1   0.00100     1000.000     0.109807E-06   0.689937E-03   4.33500
2   0.05125     19.514       0.373580E-03   0.458038E-01   5.61590
.   .           .           .             .             .
.   .           .           .             .             .
.   .           .           .             .             .
200 9.97438     0.100       0.424777      0.267581      0.168558
201 10.00000    0.100       0.423016      0.265789      0.167000

PROPERTIES OF THE INITIAL THEORETICAL PSDF:
  PSDF_INITIAL CENTRAL ANGULAR FREQUENCY (f) .. 23.5685 [Hz]
  PSDF_INITIAL CENTRAL ANGULAR FREQUENCY (WC) . 148.0853 [rad/s]
  PSDF_INITIAL DISPERSION PARAMETER (Q) ..... 0.8108 [-]
  PSDF_INITIAL STD. DEV. OF ACCELERATIONS ..... 1.2847 [m/s2]

PROPERTIES OF THE DISCRETIZED CONTENT FOR SYNTHESIS:
  TOTAL POWER (INTEGRAL OF PSDF_CONTENT) ..... 1.6477 [m2/s3]
  ROOT-MEAN-SQUARE FREQUENCY ..... 23.6529 [Hz]
  ROOT-MEAN-SQUARE CIRCULAR FREQUENCY ..... 148.6158 [rad/s]
  PERIOD CORRESPONDING TO RMS FREQUENCY ..... 0.0423 [s]
  NUMBER OF FREQUENCIES FOR SYNTHESIS ..... 1731 [#]

```

Figure 5.2: Fragment of the output file showing the echo of input parameters and the calculated properties of the initial PSDF.

5.3.2 Phase 1: Summary of Frequency-Domain Optimization

The first phase of the program aims to generate a set of `N_MOTIONS_GEN` candidate accelerograms and, for each one, optimize its PSDF over `N_ITERATIONS` cycles to minimize the difference with the target spectrum. To keep the output file concise and readable, the 2025 version of the program no longer displays the results of each individual iteration.

Instead, upon completing the optimization for a given motion, the program presents a single summary block, as shown in Figure 5.3. This block corresponds to the **best iteration cycle** found for that motion, i.e., the one that produced the lowest equivalent difference. The key values to interpret in this summary are:

- **SELECTED FROM ITERATION CYCLE:** Indicates which of the `N_ITERATIONS` produced the best result for this motion.
- **SELECTED EQUIVALENT DIFFERENCE THIS MOTION:** This is the combined error metric for that best iteration. It represents the fit quality of this particular candidate.
- **BEST-KNOWN EQUIVALENCE DIFFERENCE:** This is a tracking metric that shows the lowest error found so far among all motions generated up to this point. The program uses this value to determine which of the `N_MOTIONS_GEN` candidates is the overall best that will proceed to Phase 2.

5.3.3 Phase 2: Summary of Time-Domain Optimization

Once the program has selected the best candidate from Phase 1, the high-precision refinement process in the time domain begins. This phase is a complex iterative process, whose main working unit is the **spectral sweep**: a complete pass through all the periods of the spectrum in which the algorithm attempts to apply corrective *wavelets*.

To provide a transparent view of this optimization process, the program generates a results summary block at the end of each completed spectral sweep, as shown in Figure 5.4. This real-time report allows the user to directly observe the algorithm's convergence. In each new block, a progressive decrease in the **EQUIVALENCE DIFFERENCE THIS SWEEP** and, consequently, in the **BEST-KNOWN EQUIVALENCE DIFFERENCE** should be expected.

The iterative process of sweeps continues until the improvement in the error between one sweep and the next is less than the stagnation threshold defined by the user (`IMPROVE_THRS`). At that point, the algorithm considers that the current adjustment strategy has reached its maximum potential and, if applicable, transitions to the next “state” of the Finite State Machine to continue the refinement with a more conservative strategy. This detailed record is a valuable diagnostic tool for visualizing the effectiveness of the powerful time-domain adjustment algorithm.

```

*****
PHASE 1: GENERATION OF SEISMIC MOTIONS WITH FREQUENCY DOMAIN OPTIMIZATION
*****

= = = = =

RESULTS FOR GENERATED GROUND MOTION ..... 1 [#]
SELECTED FROM ITERATION CYCLE ..... 25 [#]
USING INITIAL SEED ..... 34641 [#]

NORMALIZED MEAN DIFFERENCE - OVERALL ..... 2.6803 [%]
NORMALIZED MEAN DIFFERENCE - TOP 5% ..... 12.6092 [%]
NORMALIZED STANDARD DEVIATION ..... 3.5117 [%]
SELECTED EQUIVALENT DIFFERENCE THIS MOTION .. 5.4951 [%]
BEST-KNOWN EQUIVALENCE DIFFERENCE ..... 5.4951 [%]

= = = = =

RESULTS FOR GENERATED GROUND MOTION ..... 2 [#]
SELECTED FROM ITERATION CYCLE ..... 24 [#]
USING INITIAL SEED ..... 76112 [#]

NORMALIZED MEAN DIFFERENCE - OVERALL ..... 2.9740 [%]
NORMALIZED MEAN DIFFERENCE - TOP 5% ..... 13.6089 [%]
NORMALIZED STANDARD DEVIATION ..... 3.8733 [%]
SELECTED EQUIVALENT DIFFERENCE THIS MOTION .. 5.9925 [%]
BEST-KNOWN EQUIVALENCE DIFFERENCE ..... 5.4951 [%]

= = = = =
. . . . .
. . . . .
. . . . .
= = = = =

RESULTS FOR GENERATED GROUND MOTION ..... 10 [#]
SELECTED FROM ITERATION CYCLE ..... 11 [#]
USING INITIAL SEED ..... 82644 [#]

NORMALIZED MEAN DIFFERENCE - OVERALL ..... 2.8874 [%]
NORMALIZED MEAN DIFFERENCE - TOP 5% ..... 13.2711 [%]
NORMALIZED STANDARD DEVIATION ..... 3.5362 [%]
SELECTED EQUIVALENT DIFFERENCE THIS MOTION .. 5.7428 [%]
BEST-KNOWN EQUIVALENCE DIFFERENCE ..... 2.7294 [%]

=====

RESULTS FOR BEST MOTION (PHASE 1) ..... 8 [#]
SELECTED FROM ITERATION CYCLE ..... 23 [#]
USING INITIAL SEED ..... 49596 [#]

NORMALIZED MEAN DIFFERENCE - OVEALL ..... 1.3522 [%]
NORMALIZED MEAN DIFFERENCE - TOP 5% ..... 6.2700 [%]
NORMALIZED STANDARD DEVIATION ..... 1.7215 [%]
EQUIVALENT DIFFERENCE (BEST OF PHASE 1) ..... 2.7294 [%]

```

Figure 5.3: Fragment of the output file showing the summary of results for a motion generated in Phase 1.

```

*****
PHASE 2: TIME-DOMAIN ADJUSTMENT USING CORRECTIVE WAVELETS
*****

-----

ENTERING OPTIMIZATION STATE ..... 1 [#]
INITIAL WAVELET STRENGTH ..... 0.4000 [-]
WAVELET CYCLES ..... 4 [#]
ADAPTIVE DECAY FACTOR ..... 0.7500 [-]

=====

RESULTS FOR SWEEP OF STATE ..... 1 OF 1 [#]

NORMALIZED MEAN DIFFERENCE - OVERALL ..... 1.2682 [%]
NORMALIZED MEAN DIFFERENCE - TOP 5% ..... 4.8858 [%]
NORMALIZED STANDARD DEVIATION ..... 1.3759 [%]
EQUIVALENCE DIFFERENCE THIS SWEEP ..... 2.2157 [%]
BEST-KNOWN EQUIVALENCE DIFFERENCE ..... 2.2157 [%]

=====
. . . . .
. . . . .
. . . . .
=====

RESULTS FOR SWEEP OF STATE ..... 1 OF 4 [#]

NORMALIZED MEAN DIFFERENCE - OVERALL ..... 0.8001 [%]
NORMALIZED MEAN DIFFERENCE - TOP 5% ..... 2.7064 [%]
NORMALIZED STANDARD DEVIATION ..... 0.7935 [%]
EQUIVALENCE DIFFERENCE THIS SWEEP ..... 1.2740 [%]
BEST-KNOWN EQUIVALENCE DIFFERENCE ..... 1.2740 [%]

-----

ENTERING OPTIMIZATION STATE ..... 5 [#]
INITIAL WAVELET STRENGTH ..... 0.0800 [-]
WAVELET CYCLES ..... 7 [#]
ADAPTIVE DECAY FACTOR ..... 0.9500 [-]

=====

RESULTS FOR SWEEP OF STATE ..... 1 OF 5 [#]

NORMALIZED MEAN DIFFERENCE - OVERALL ..... 0.8006 [%]
NORMALIZED MEAN DIFFERENCE - TOP 5% ..... 2.6892 [%]
NORMALIZED STANDARD DEVIATION ..... 0.7844 [%]
EQUIVALENCE DIFFERENCE THIS SWEEP ..... 1.2663 [%]
BEST-KNOWN EQUIVALENCE DIFFERENCE ..... 1.2663 [%]

```

Figure 5.4: Fragment of the output file showing the progress of the adjustment in Phase 2, with the results after each spectral sweep.

5.3.4 Detailed Analysis of Final Results

Upon completion of the two optimization phases, the program presents a final section that comprehensively summarizes the properties of the definitive artificial earthquake.

Comparative Summary and Motion Characteristics

The main results block (see Figure 5.5) encapsulates the most relevant information from the simulation. The first part presents a direct comparison of the difference metrics before and after Phase 2, quantifying the significant improvement achieved by the time-domain refinement.

Following this, a comprehensive set of parameters characterizing the final earthquake is detailed, including both kinematic peaks (PGA, PGV, PGD, and PFA) and a full range of intensity measures: Arias Intensity (I_A), Potential Destructive Impulse (I_D), Cumulative Absolute Velocity (CAV), Housner's Spectral Intensity (HI), and Significant Duration (D_{5-95}). The theoretical basis for each of these intensity metrics is described in detail in Section 3.3.

```
#####
OVERALL BEST GROUND MOTION
#####

RESULTS FOR GENERATED GROUND MOTION NUMBER ..      8 [#]
SELECTED FROM FREQUENCY-DOMAIN ITERATION ....      23 [#]
USING INITIAL RANDOM SEED .....                  49596 [#]

METRICS AFTER FREQUENCY-DOMAIN OPTIMIZATION WITH SMOOTH PSDF:
NORMALIZED MEAN DIFFERENCE - OVERALL .....      1.3522 [%]
NORMALIZED MEAN DIFFERENCE - TOP 5% .....      6.2700 [%]
NORMALIZED STANDARD DEVIATION .....            1.7215 [%]
EQUIVALENT DIFFERENCE (BEST OF PHASE 1) ...      2.7294 [%]

METRICS AFTER TIME-DOMAIN ADJUSTMENT WITH CORRECTIVE WAVELETS:
NORMALIZED MEAN DIFFERENCE - OVERALL .....      0.8006 [%]
NORMALIZED MEAN DIFFERENCE - TOP 5% .....      2.6892 [%]
NORMALIZED STANDARD DEVIATION .....            0.7844 [%]
EQUIVALENT DIFFERENCE (BEST OF PHASE 2) ...      1.2663 [%]

PEAK GROUND ACCELERATION (FINAL) .....          4.3310 [m/s2]
AT TIME .....                                0.6640 [s]
PEAK GROUND ACCELERATION (BEFORE SCALE) .....    3.6039 [m/s2]
AT TIME .....                                0.6640 [s]
MAXIMUM GROUND VELOCITY .....                  0.3299 [m/s]
MAXIMUM GROUND DISPLACEMENT .....              0.5146 [m]

MAXIMUM PEAK FLOOR ACCELERATION (PFA) .....      9.4891 [m/s2]
OCCURRING AT PERIOD .....                      0.1946 [s]

ARIAS INTENSITY (IA) .....                     1.7390 [m/s]
DESTRUCTIVE POTENTIAL (ID+) .....               1.0674 [m/s]
DESTRUCTIVE POTENTIAL (ID-) .....               0.6716 [m/s]
VERIFICATION (ID+ + ID-) .....                 1.7390 [m/s]
NET ENERGY IMPULSE (ID+ - ID-) .....           0.3958 [m/s]
CUMULATIVE ABSOLUTE VELOCITY (CAV) .....         9.5430 [m/s]
HOUSNER INTENSITY (HI) .....                   1.0117 [m]
SIGNIFICANT DURATION (D5-95) .....              9.2880 [s]
```

Figure 5.5: Fragment of the output file showing the results block, with the comparison of metrics and the final properties of the generated earthquake.

Spectral Analysis and Graphical Visualization

Standardization of the Final Spectrum: Calculation Fidelity vs. Presentation Clarity. It is fundamental to distinguish between the high numerical resolution that the algorithm may require internally to achieve “super-compatibility” and the resolution used for the final presentation of the results. The spectrum of a signal adjusted with high precision may exhibit small oscillations or “computational noise,” which are artifacts of the optimization and not flaws in the fit.

Furthermore, an independent reviewer analyzing the final accelerogram will do so by calculating its spectrum with an arbitrary number of points, different from that used in the adjustment. To reconcile this duality, the program implements a post-processing stage (subroutine `RESAMPLE_SPECTRUM`) that standardizes the output spectrum to a fixed number of points. The objective is to decouple the calculation resolution from that of the presentation, providing a final representation that is clear, smooth, and consistently sized, which faithfully reflects the properties of the generated accelerogram.

The program then presents the complete response spectrum table for the primary damping, now standardized. This table allows for a detailed numerical verification of the fit at each point of the spectrum. Tables for the response spectra with secondary damping ratios are also presented. These tables are shown in Figure 5.6.

STANDARDIZED RESPONSE SPECTRUM FOR PRIMARY DAMPING = 0.050:								
#	PERIOD [s]	FREQ. [Hz]	TARGET SV [m/s]	CALC SV [m/s]	DIFFER [%]	DISPL.SPECT [m]	ACCEL.SPECT [m/s ²]	TIME MAX [s]
1	0.01000	100.00	0.80273E-02	0.84768E-02	5.6	0.13491E-04	5.3261	0.666
2	0.01035	96.59	0.83923E-02	0.89785E-02	7.0	0.14794E-04	5.4489	0.667
3	0.01072	93.29	0.87719E-02	0.93574E-02	6.7	0.15963E-04	5.4851	0.658
4	0.01110	90.11	0.90351E-02	0.96921E-02	7.3	0.17118E-04	5.4874	0.658
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
197	9.01102	0.11	0.32882	0.32708	-0.5	0.46908	0.22807	12.684
198	9.32930	0.11	0.30825	0.30745	-0.3	0.45650	0.20706	12.944
199	9.65883	0.10	0.28426	0.28515	0.3	0.43834	0.18549	13.144
200	10.00000	0.10	0.26579	0.27769	4.5	0.44196	0.17448	9.160

STANDARDIZED RESPONSE SPECTRUM FOR SECONDARY DAMPING = 0.400:								
#	PERIOD [s]	FREQ [Hz]	PSA [m/s ²]	PSV [m/s]	SD [m]	TIME MAX PSV [s]		
1	0.01000	100.00	4.3122	0.68630E-02	0.10923E-04	0.666		
2	0.01035	96.59	4.3393	0.71501E-02	0.11782E-04	0.666		
3	0.01072	93.29	4.3667	0.74494E-02	0.12708E-04	0.666		
4	0.01110	90.11	4.3362	0.76586E-02	0.13527E-04	0.666		
.	.	.	.	.	.	.		
.	.	.	.	.	.	.		
.	.	.	.	.	.	.		
197	9.01102	0.11	0.11688	0.16763	0.24041	8.020		
198	9.32930	0.11	0.10906	0.16194	0.24044	8.032		
199	9.65883	0.10	0.10131	0.15574	0.23942	8.040		
200	10.00000	0.10	0.95082E-01	0.15133	0.24085	8.048		

Figure 5.6: Final response spectrum table for the primary damping (top) and for a secondary damping (bottom).

Visually, the excellent compatibility achieved can be seen in the graphical comparisons generated by the post-processing script `process_earthquake_output_2025.py`. Figures 5.7 to 5.9 show the pseudo-acceleration, pseudo-velocity, and displacement spectra, respectively, in both linear and logarithmic scales.

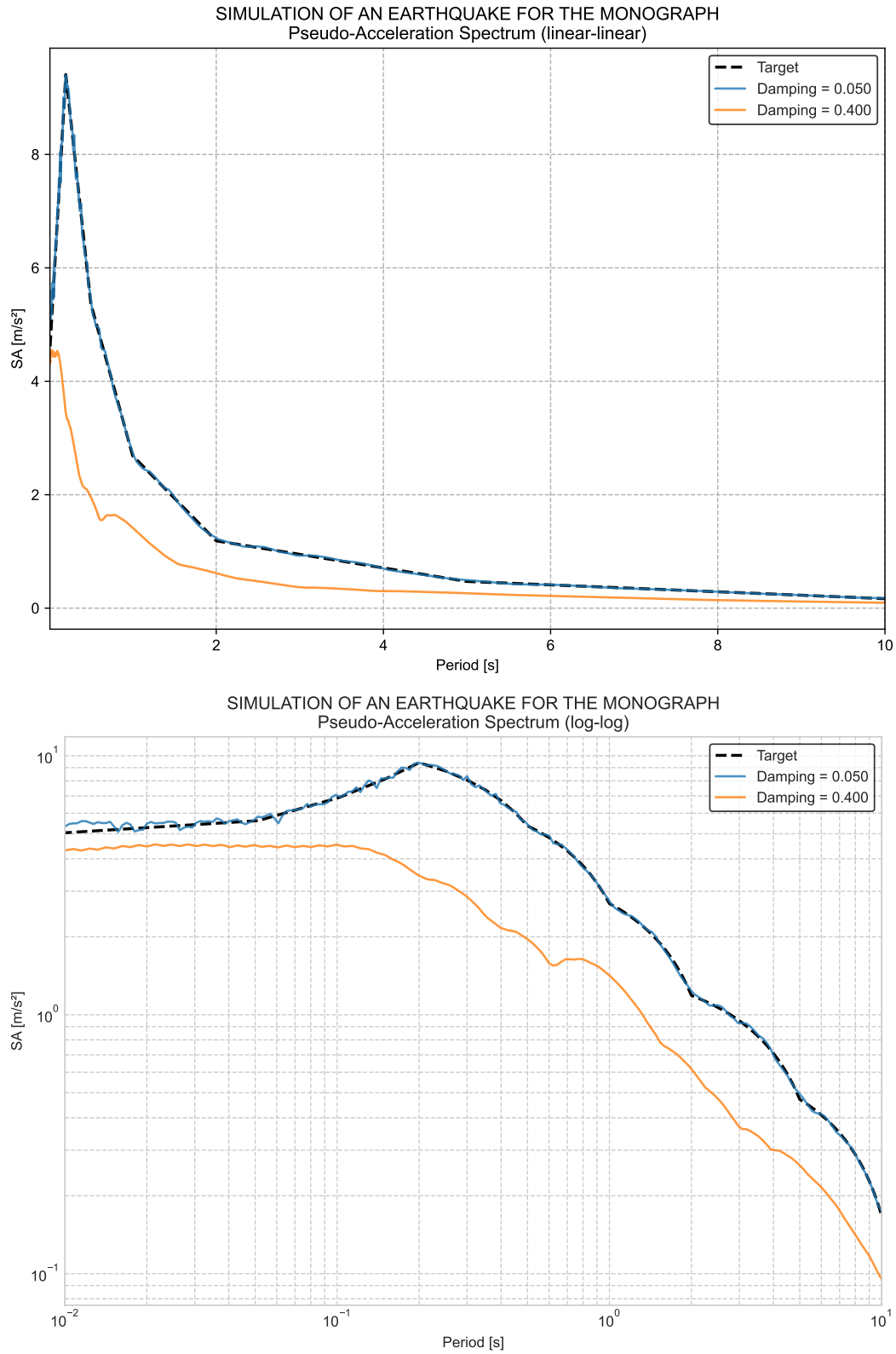


Figure 5.7: Comparison of Pseudo-Acceleration (PSA) Spectra in linear scale (top) and log-log scale (bottom).

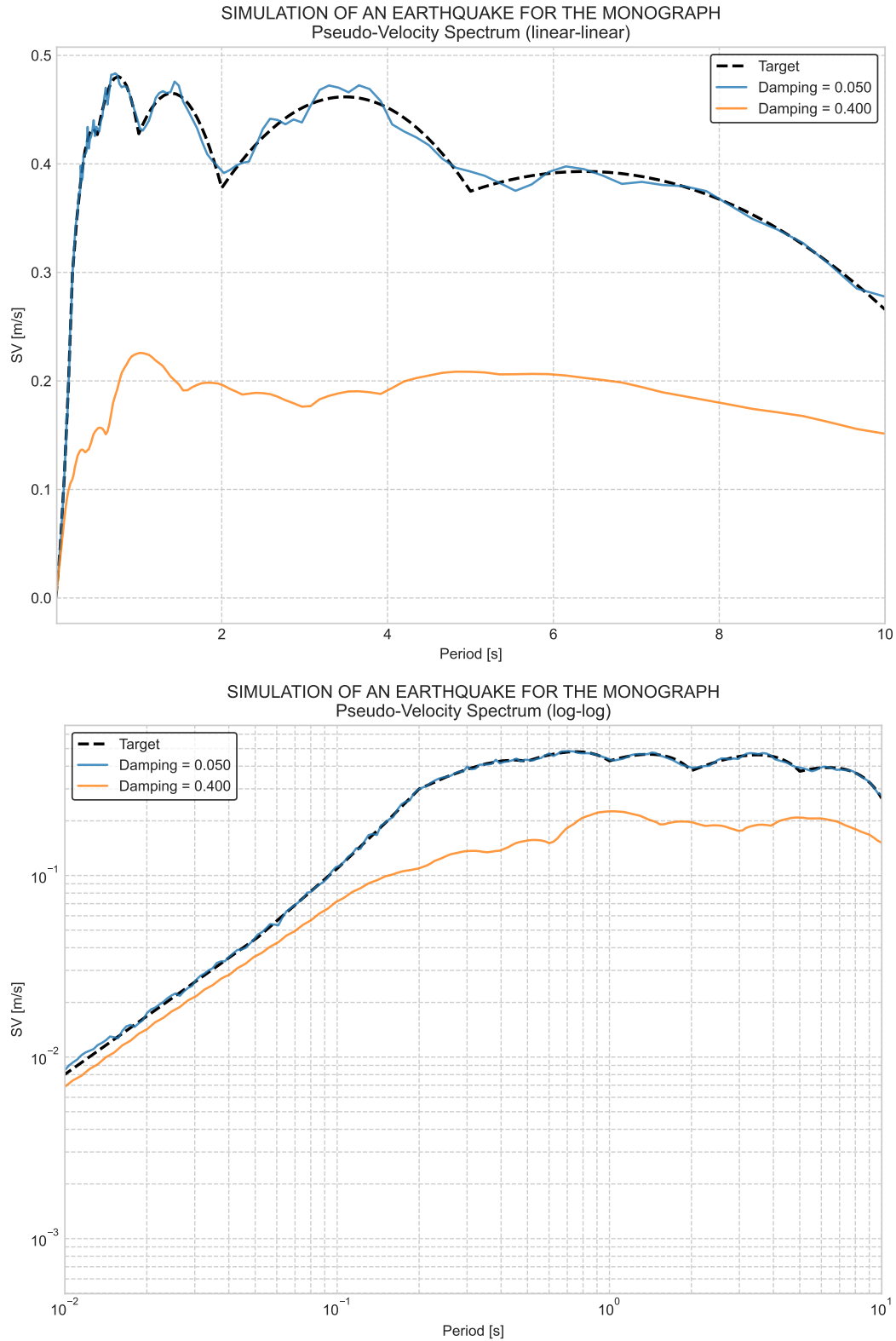


Figure 5.8: Comparison of Pseudo-Velocity (PSV) Spectra in linear scale (top) and log-log scale (bottom).

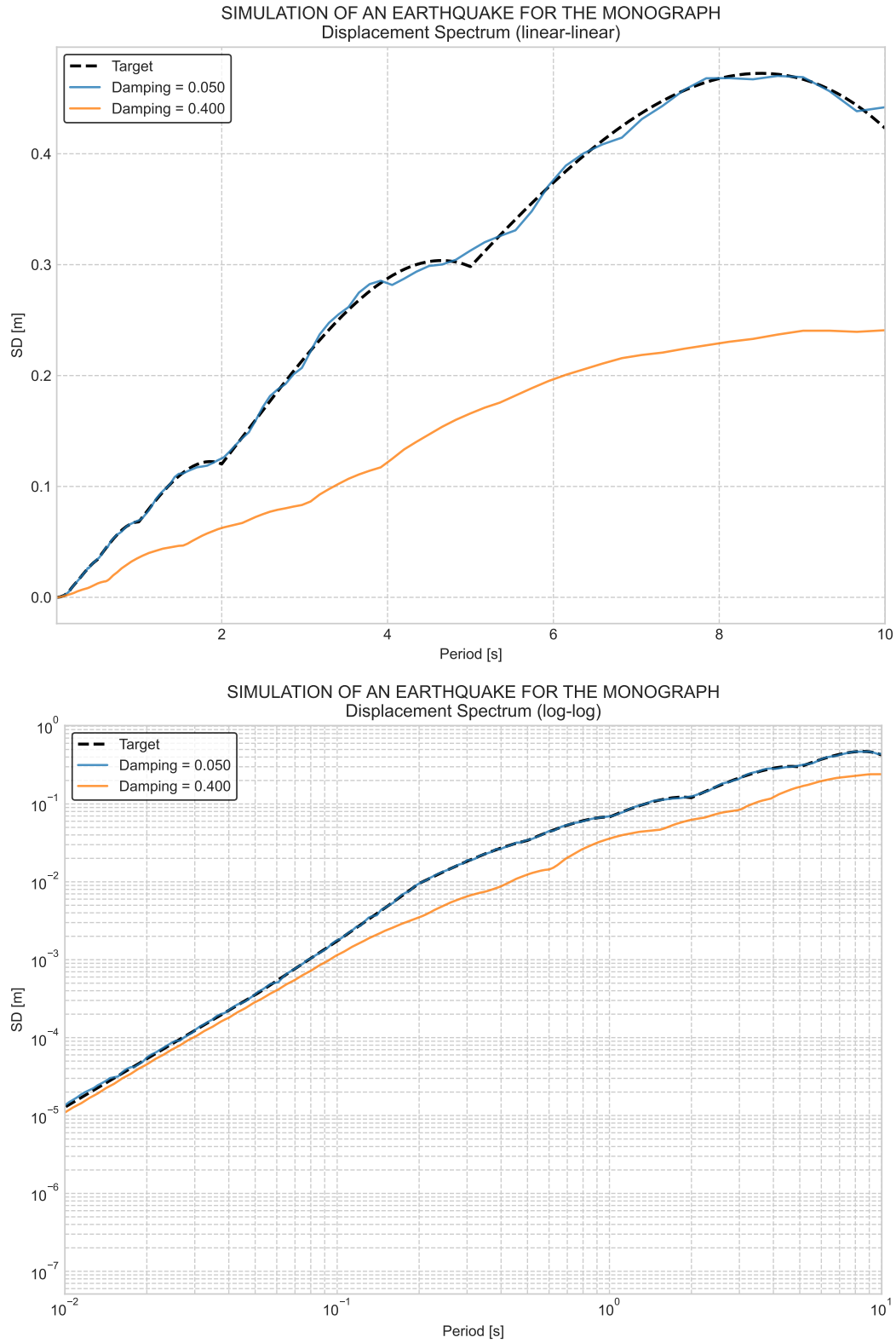


Figure 5.9: Comparison of Displacement (SD) Spectra in linear scale (top) and log-log scale (bottom).

Time Histories

The output file also includes a table with the complete time histories of the final motion (acceleration, velocity, and displacement), an excerpt of which is shown in Figure 5.10. Additionally, the post-processing script visualizes this information in three separate plots (Figures 5.11 to 5.13), which allow for a detailed analysis of the kinematic evolution of the generated earthquake. In these plots, the time interval corresponding to the Significant Duration (D_{5-95}) is highlighted with a shaded area, allowing the user to visually identify the most intense shaking phase of the motion.

NUMBER OF POINTS IN TIME HISTORY				5001 [#]
TIME INCREMENT				0.0040 [s]
TOTAL DURATION				20.0000 [s]
#	TIME [s]	DISPLACEMENT [m]	VELOCITY [m/s]	ACCELERATION [m/s ²]
1	0.0000	0.00000	0.00000	-0.116656E-01
2	0.0040	-0.439492E-07	-0.219746E-04	0.678316E-03
3	0.0080	-0.728748E-07	0.751182E-05	0.140649E-01
4	0.0120	0.165090E-06	0.111471E-03	0.379146E-01
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
4998	19.9880	-0.142983E-05	0.224574E-03	-0.940347E-02
4999	19.9920	-0.632513E-06	0.174084E-03	-0.158412E-01
5000	19.9960	-0.960096E-07	0.941673E-04	-0.241173E-01
5001	20.0000	0.923345E-07	0.477526E-08	-0.229639E-01

Figure 5.10: Excerpt from the time series table in the output file.

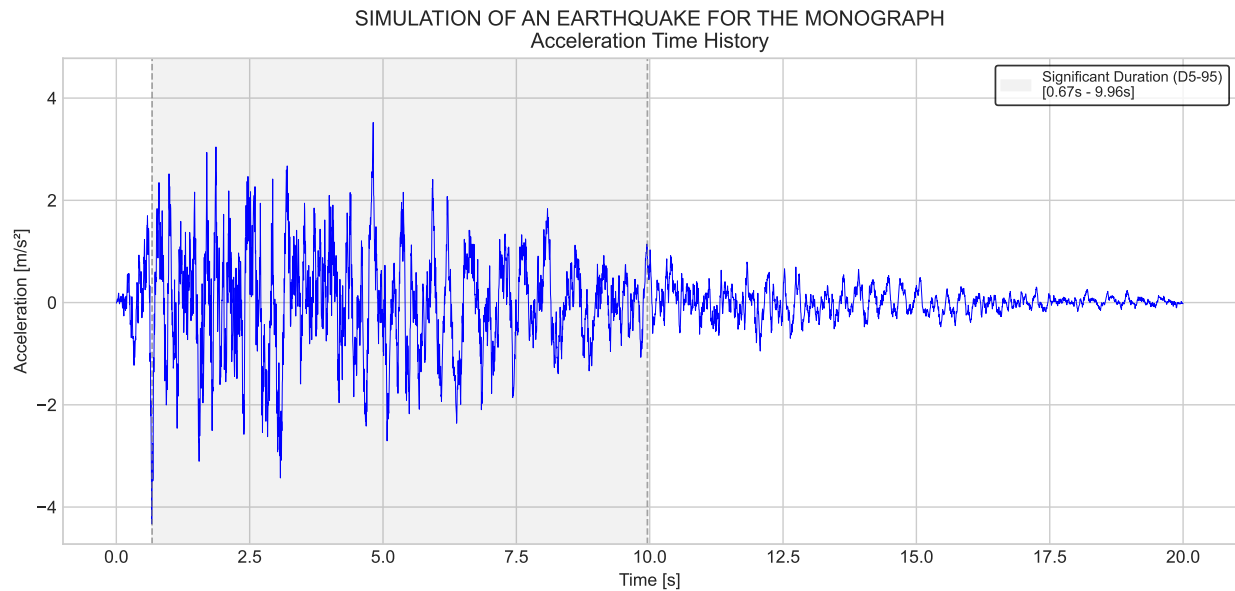


Figure 5.11: Acceleration time history of the generated motion ($a_g(t)$).

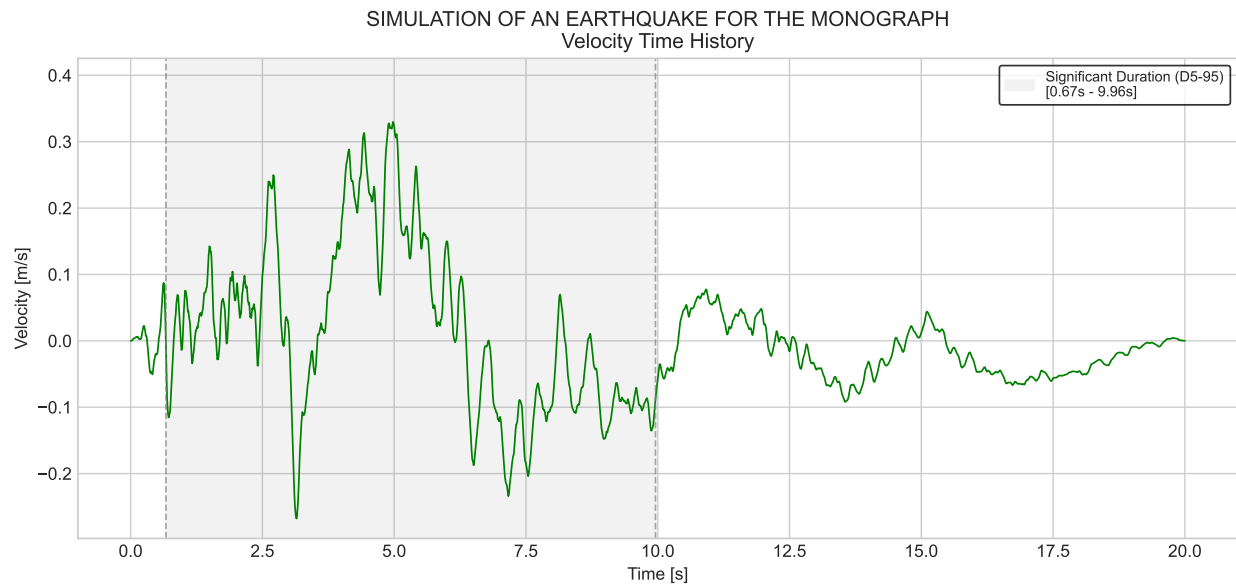


Figure 5.12: Velocity time history of the generated motion ($v_g(t)$).

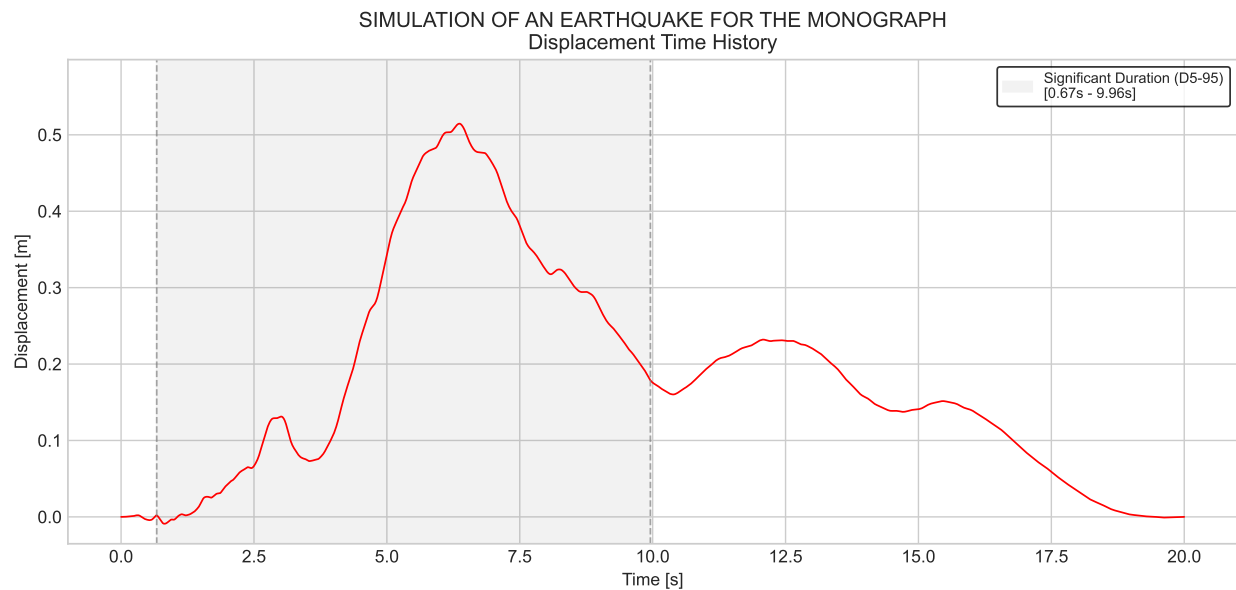


Figure 5.13: Displacement time history of the generated motion ($d_g(t)$).

Energy Evolution: The Husid Plot

In addition to the kinematic histories, the program generates data to visualize the **Normalized Husid Plot**, as shown in Figure 5.14. This curve represents the accumulation of seismic energy (proportional to $\int a_g^2(t)dt$) over time. It is a very powerful diagnostic tool:

- The **slope** of the curve indicates the rate of energy release. Steep slopes correspond to the most intense phases of the earthquake.
- It allows for **direct visualization** of the Significant Duration (D_{5-95}), which corresponds to the time interval it takes for the curve to go from 5% to 95% of its final value.

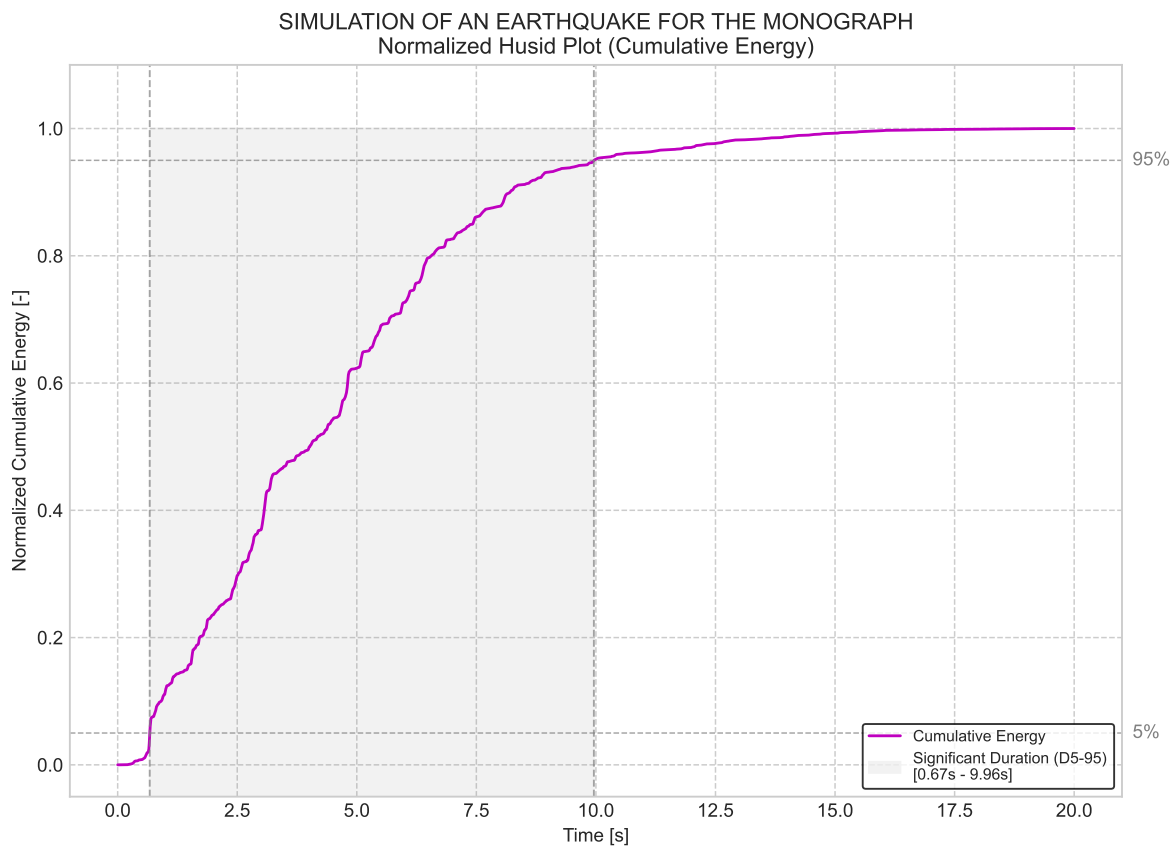


Figure 5.14: Example of the Normalized Husid Plot, showing the progressive accumulation of energy. The horizontal lines mark the 5% and 95% thresholds used to define the Significant Duration.

5.4 Advanced Interpretation and Diagnostic Guide

Beyond simply running the program, a correct interpretation of the results and the ability to diagnose problems are crucial for professional use of the tool. This section addresses frequently asked questions and offers guidelines for troubleshooting and obtaining the highest quality results.

5.4.1 The Utility of Secondary Damping Ratios

Although the generation and adjustment process of the artificial earthquake is based exclusively on the **first damping ratio** in the provided list (the “primary damping”), the program calculates and reports the response spectra for all specified damping ratios. The utility of this feature is manifold:

- **Sensitivity Analysis:** It allows for a quick evaluation of how the spectral demand changes with variations in structural damping.
- **Design with Energy Dissipators:** It offers a preliminary linear estimate of the response reduction that could be obtained by adding dissipation systems that increase the structure’s effective damping.
- **Verification for Multiple Limit States:** It facilitates the analysis of structures that may exhibit different damping levels depending on the level of damage.
- **Academic Research:** It provides a complete and consistent dataset for comparative studies, all in a single program run.

It is important to emphasize that the generated earthquake is compatible only with the spectrum of the primary damping. The secondary spectra are an informational result of how that specific signal affects systems with a different level of damping.

5.4.2 Guide to Achieving “Super-Compatibility”

The 2025 version of `earthquake_2025` is designed to achieve very high levels of spectral compatibility. To maximize the quality of the fit, the user can act on several key parameters, especially when facing a complex target spectrum:

1. **Ensure PGA Consistency:** As detailed in Subsection 5.2.2, the specified PGA value must be consistent with the target spectrum’s ordinate at $T = 0$. A discrepancy here is the main source of poor fitting in the high-frequency zone.
2. **Increase Frequency Richness:** A smaller value for the `RfreqInc` parameter (e.g., 0.001) generates a synthetic signal composed of more sinusoids, giving the algorithm greater flexibility to adjust fine details of the target spectrum.

3. **Increase Spectrum Resolution:** The `NPERIOD` parameter defines into how many points the spectrum is discretized for its calculation and adjustment. A larger number (e.g., 300 or 500) provides the correction algorithm with a more detailed view of the spectrum's shape, which can help achieve a finer fit, especially if the target spectrum has rapid variations.
4. **Increase the Number of Iterations:** Increasing `NITERA` (e.g., to 50 or 100) in Phase 1 gives the algorithm more opportunities to refine the PSDF and find a better starting point for Phase 2.
5. **Generate Multiple Motions:** Increasing `NMOTION` (e.g., to 50 or 80) explores different random phase sequences. Since some sequences are inherently “easier” to fit than others, generating more candidates increases the probability of finding one that converges to a very low error.
6. **Reduce the Improvement Threshold:** The `IMPROVE_THRS` parameter controls the termination of Phase 2. A smaller value (e.g., 0.001%) will force the algorithm to perform more spectral sweeps in the time domain, squeezing out every last possible improvement, albeit at the cost of longer computation time.

5.4.3 Diagnosing Common Errors

- **Analysis of Poor Convergence:** If the “Equivalent Difference” does not decrease satisfactorily in Phase 1, the most frequent cause is a **physically unrealistic target spectrum**. A spectrum with extremely steep slopes or very sharp cusps is difficult to fit, as a band-limited stochastic process, like the one generated, tends to produce spectra with smoother variability [19].
- **STOP Error Messages:** The program performs a series of validations on the input data to prevent nonsensical simulations from running. A `STOP 'INVALID...'` message is a deliberate interruption indicating a fatal configuration error that must be corrected by the user. The error message is usually self-explanatory and points to the erroneous parameter or condition, such as: `N_PTS_TARGET_SPECT < 3` (a minimum of 3 points are needed to define the spectrum), periods in the target spectrum that are not strictly increasing, or envelope parameters that are physically inconsistent (e.g., `PARAM_ALPHA_EXP <= PARAM_BETA_EXP` in the exponential envelope).
- ***WARNING* Messages:** A warning does not stop the program's execution but informs the user of an anomalous condition that has been detected and managed internally. The most common warning is `*WARNING* PSDF ... IS NEGATIVE`. This can occur in the first iterations of Phase 1 if the spectral adjustment algorithm, in its attempt to correct a large discrepancy, overestimates the correction. The program handles this by taking the absolute value of the PSDF. If this warning appears only in the early iterations, it is not a major cause for concern. However, if the warning persists in advanced iterations, it may be a symptom of instability in the convergence.

Chapter 6

Computational Methodology of earthquake_2025

6.1 General Simulation Flow: An Overview of the Process

The `earthquake_2025` program implements a robust and sequential computational process for the generation of accelerograms. Understanding this flow is essential for interpreting the results and diagnosing the simulation's behavior. The process can be summarized in the following logical steps:

1. **Reading and Validation of Input Parameters:** The program begins by reading all the parameters defined by the user in the input file. A comprehensive validation is performed to ensure the consistency and physical viability of the data.
2. **Calculation of the Initial PSDF:** From the target response spectrum (either PSA or PSV) provided by the user, the program calculates a first estimate of the acceleration's Power Spectral Density Function (PSDF). This is an inverse problem that constitutes the starting point of the stochastic process.
3. **Motion Simulation Loop:** A main loop is initiated, which repeats for each motion to be generated (controlled by the `N_MOTION_GEN` (NMOTION) parameter). For each new motion, a new sequence of random phases is generated, ensuring that each accelerogram is a statistically independent realization.
4. **Phase 1 - Frequency-Domain Optimization:** Within the loop for each motion, an iterative loop is initiated (controlled by `N_ITERATIONS` (NITERA)) that forms the core of the frequency-domain optimization.
 - (a) **Accelerogram Synthesis:** A time-history acceleration signal is generated by the superposition of sinusoids, using the current PSDF and the motion's random phase sequence.

- (b) **Signal Processing:** The synthetic accelerogram is modulated by the intensity envelope function and subjected to a baseline correction.
 - (c) **Evaluation and Correction:** The response spectrum is calculated, the error with respect to the target spectrum is quantified, and the PSDF is adjusted for the next iteration.
5. **Selection of the Best Candidate:** Upon completion of Phase 1 for all motions, the program selects the accelerogram that has shown the best overall fit.
6. **Phase 2 - Time-Domain Refinement:** This best candidate earthquake is then subjected to the new time-domain optimization phase, which is governed by a **Finite State Machine**. The program progresses through different adjustment strategies, combining an optimal global scaling of the signal with the addition of corrective *wavelets* to refine the fit until “super-compatibility” is achieved.

6.2 Preprocessing the Target Spectrum: Densification for Fidelity

Before initiating any calculations, the program performs a critical preprocessing step on the target spectrum provided by the user, managed by the `DENSIFY_SPECTRUM_LINEARLY` subroutine. The goal of this step is to ensure a high-fidelity internal representation, especially for code-based design spectra, which are often defined by piecewise-linear segments on an Acceleration vs. Period scale.

The program’s fitting algorithm internally uses logarithmic interpolation to evaluate the target spectrum at the calculation points. If the spectrum provided by the user is “sparse” (defined by a small number of points, such as the vertices that define the straight-line segments of a code-based spectrum), a direct logarithmic interpolation between these vertices would not reproduce the straight segments but would instead curve them, “cutting the corners.” This would introduce a systematic error that would prevent an accurate fit from being achieved.

To avoid this, the densification subroutine adds intermediate points to the target spectrum using **linear interpolation** in the Acceleration-Period domain. This process ensures that all straight-line segments of the original spectrum are represented internally with a high density of points, eliminating interpolation error and allowing the fitting algorithm to work against a faithful representation of the true shape of the design spectrum.

6.3 The Starting Point: Generation of the Initial PSDF

The problem of generating an accelerogram compatible with a spectrum is, at its core, an inverse problem. The program does not know a priori the frequency distribution of energy (the PSDF) that will produce an accelerogram whose response spectrum matches the target spectrum. Therefore, the first crucial step is to estimate a plausible initial PSDF.

This task is carried out by the `COMPUTE_PSDF_FROM_SV` subroutine. The method is based on the relationships established in the theory of random processes. The fundamental relationship, as detailed in Monograph IS-10 [6] (Equations 4.14 and 4.23), connects the value of the response spectrum at a frequency ω_n , $S(\omega_n)$, with the standard deviation of the oscillator's response, $\sigma_u(\omega_n)$, through a **peak factor**, ζ_p :

$$S(\omega_n) = \zeta_p \cdot \sigma_u(\omega_n)$$

The original methodology, documented in said monograph, used a variable peak factor based on Vanmarcke's theory [60]. While theoretically advanced, this approach proved to be numerically sensitive and of marginal benefit within an iterative scheme. Therefore, the 2025 version adopts a simpler and more robust approach, using a **constant peak factor of 2.5**, an industry-standard value. This simplification makes the calculation of the initial PSDF faster, more numerically stable, and conceptually clearer.

The variance of the response, in turn, is directly related to the integral of the excitation's PSDF. By iteratively inverting this relationship, the subroutine “solves for” the value of the PSDF at each frequency point to satisfy the target spectrum, taking into account the cumulative contribution from lower frequencies. It is important to note that this change only affects the initial estimate (iteration 0); the accuracy of the final result is not compromised, as the subsequent correction algorithms converge to the correct PSDF regardless of the initial guess.

6.4 Phase 1: Frequency-Domain Optimization

The core of the program's first phase is an iterative process designed to adjust the signal's Power Spectral Density Function (PSDF) until the response spectrum of the resulting accelerogram reasonably matches the target spectrum.

6.4.1 Accelerogram Synthesis and Modulation

Each iteration begins with the generation of a trial accelerogram.

Superposition of Sinusoids and Random Phases The mathematical model for the synthesis of a stationary accelerogram, $a_{st}(t)$, is a superposition of a large number of sinusoidal waves [52], as described in Equation 4.2 of Monograph IS-10 [6]:

$$a_{st}(t) = \sum_{i=1}^N A_i \sin(\omega_i t + \phi_i) \quad (6.1)$$

where the components of the sum are:

- ω_i : Are the N discrete angular frequencies into which the spectrum is divided.
- ϕ_i : Are the N phase angles, generated as random variables with a uniform distribution in the interval $[0, 2\pi]$. The randomness of the phases is what ensures that each generated accelerogram is a unique and unpredictable realization of the stochastic process. This task is performed by the `GENERATE_RANDOM_PHASES` subroutine.

- A_i : Are the N amplitudes of each sinusoidal component. The amplitude of each harmonic is not an arbitrary value but is directly related to the process's energy in that frequency band. This energy is quantified by the acceleration's **Power Spectral Density Function (PSDF)**, $G_a(\omega_i)$, whose units are [energy per unit mass, per unit frequency], typically $(\text{m/s}^2)^2/(\text{rad/s})$. The relationship is given by:

$$A_i = \sqrt{2G_a(\omega_i)\Delta\omega_i} \quad (6.2)$$

where $\Delta\omega_i$ is the width of the frequency band associated with component i . This equation shows how the PSDF, which is a statistical description in the frequency domain, is used to generate the amplitudes of a deterministic signal in the time domain.

Signal Modulation with Envelope Functions. The stationary signal $a_{st}(t)$ generated by Equation 6.1 represents shaking of constant intensity over time, which is not realistic. To impart the transient character of a real earthquake (a build-up phase, a strong-shaking phase, and a decay phase), this signal must be modulated by a deterministic intensity function, $I(t)$. The final accelerogram, $a(t)$, is the product of both:

$$a(t) = I(t) \cdot a_{st}(t) \quad (6.3)$$

This operation is performed by the `APPLY_ENVELOPE` subroutine. The program offers four types of envelope functions, selected via the `ENV_TYPE` input parameter. Figure 6.1 illustrates these shapes. Their mathematical formulations are detailed below.

- **Type 1: Rectangular.** This is the simplest option, where the intensity is constant for the entire duration of the signal. It is equivalent to applying no modulation. $I(t) = 1.0$ for $0 \leq t \leq T_d$.
- **Type 2: Trapezoidal.** This envelope, defined by the parameters `T_RISE_ENV` (T_r) and `T_PLATEAU_ENV` (T_p), models a linear rise, a plateau of constant intensity, and a linear decay phase. Let $T_f = T_r + T_p$ be the time at the end of the plateau:

$$I(t) = \begin{cases} t/T_r & \text{if } 0 \leq t < T_r \\ 1.0 & \text{if } T_r \leq t < T_f \\ (T_d - t)/(T_d - T_f) & \text{if } T_f \leq t \leq T_d \end{cases} \quad (6.4)$$

- **Type 3: Exponential.** This function, controlled by parameters `PARAM_A0_EXP` (a_0), `PARAM_ALPHA_EXP` (α), and `PARAM_BETA_EXP` (β), allows for modeling a smooth and continuous rise and fall, widely used in seismological literature to represent the character of real earthquakes [34].

$$I(t) = a_0 \cdot (e^{-\beta t} - e^{-\alpha t}) \quad (6.5)$$

For the function to have a valid envelope shape, the condition must be met: $\alpha > \beta > 0$.

Seismic Intensity Envelope Functions (earthquake)

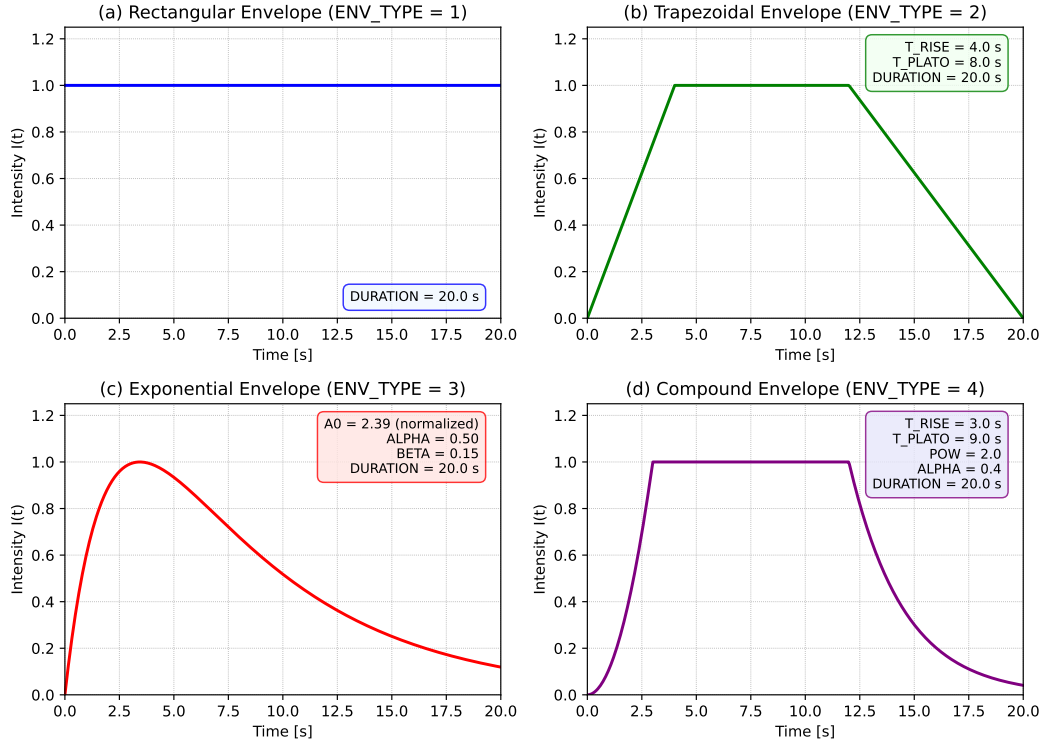


Figure 6.1: Examples of the four intensity envelope functions implemented in the program, along with the parameters used for their generation.

- **Type 4: Compound.** This is a hybrid envelope that combines a power-law rise, a plateau, and an exponential decay. It is defined by the parameters T_{RISE_ENV} (T_r), $T_{PLATEAU_ENV}$ (T_p), $PARAM_POW_CMPD$ (n), and $PARAM_ALPHA_EXP$ (α_{decay}). Again, let $T_f = T_r + T_p$:

$$I(t) = \begin{cases} (t/T_r)^n & \text{if } 0 \leq t < T_r \\ 1.0 & \text{if } T_r \leq t < T_f \\ e^{-\alpha_{decay}(t-T_f)} & \text{if } T_f \leq t \leq T_d \end{cases} \quad (6.6)$$

The choice of an envelope and its associated parameters should be based on the expected characteristics of the seismicity to be simulated (e.g., magnitude, fault distance), as the envelope shape has a direct influence on engineering metrics such as Arias Intensity and Significant Duration.

6.4.2 Iterative Evaluation and Correction

Once a trial accelerogram for the current iteration has been generated and processed, the evaluation and correction cycle begins.

Response Spectrum Calculation (SPECT and MATAB). The first step is to calculate its response spectrum to compare it with the target spectrum. This task is performed by the **SPECT** subroutine. The implemented method solves the SDOF equation of motion (Equation 4.2) using a direct integration scheme in the state space. The solution for the state vector (displacement and velocity) at step $i + 1$ is obtained from step i through the recurrence:

$$\begin{Bmatrix} u \\ \dot{u} \end{Bmatrix}_{i+1} = [A] \begin{Bmatrix} u \\ \dot{u} \end{Bmatrix}_i + [B] \begin{Bmatrix} a_{g,i} \\ a_{g,i+1} \end{Bmatrix}$$

The state transition matrix $[A]$ and the load application matrix $[B]$ are calculated analytically by the **MATAB** subroutine, assuming a linear variation of acceleration within the time step. A key improvement incorporated in the 2022 version is that **SPECT** can perform this integration using **time sub-steps**, that is, by dividing the original time step of the accelerogram into several smaller sub-intervals. This significantly increases the accuracy of the Duhamel integral solution, especially for short-period oscillators where the original time step might be too large.

Quantification of the Difference (CALCULATE_WEIGHTED_DIFFERENCE_METRICS). The comparison between the calculated spectrum and the target spectrum cannot be based on a simple difference. To address this, the program implements a robust approach in the **CALCULATE_WEIGHTED_DIFFERENCE_METRICS** subroutine. The method is based on the **normalized absolute difference**, which weights the difference at each point by the maximum value of the target spectrum. From this set of normalized differences, three key statistical metrics are calculated:

1. The **mean** of the differences (“Normalized Mean Difference”), which indicates the average deviation of the fit.
2. The **standard deviation** of the differences (“Normalized Standard Deviation”), which measures the variability of the fit across the spectrum.
3. The **average of the top 5%** of the differences (“Top 5% Mean Difference”), which replaces the volatile “maximum difference” as a robust indicator of the largest discrepancies.

Finally, these three metrics are combined using a weighted sum to obtain an **equivalent difference**, which is the value that guides the selection of the best result throughout the entire process.

The PSDF Correction Algorithm (SMOOTH_PSDF_FOR_ITERATION). The correction of the PSDF in each iteration is the most delicate step in phase 1. The original method (Equation 4.27 of Monograph IS-10 [6]) applies a pointwise correction: $G_{k+1}(\omega_j) = G_k(\omega_j) \cdot (S_{obj}(\omega_j)/S_{calc}(\omega_j))^2$. While conceptually straightforward, this method can be numerically unstable and prone to creating spurious peaks. Similar, though more refined, approaches were the basis for early earthquake generation programs like PSEQGN [53] and SIMQKE [29], which are founded on the theory of random vibrations developed by Vanmarcke [60].

Since the 2022 version, the `SMOOTH_PSDF_FOR_ITERATION` subroutine has implemented a significantly more sophisticated and robust multi-stage strategy than all previous ones to ensure stable convergence:

1. **Smoothed Correction:** Instead of applying the correction ratio point by point, a “smoothed” ratio is calculated using a weighted moving average that considers the influence of neighboring frequencies. This prevents a single spectral discrepancy from generating an unrealistic peak in the PSDF, allowing for the coordinated correction of broader spectral regions.
2. **Limitation of Maximum Change:** The change applied to the PSDF at a single point in a single iteration is strictly limited in the program by preset control factors (`MAX_CHANGE_FACTOR` and `RELAXATION_FACTOR`). This acts as a numerical damper that prevents excessive changes and violent oscillations of the solution between iterations.
3. **Global Energy Adjustment:** After defining the new shape of the PSDF, its total power (the integral of the PSDF, representing the acceleration variance) is calculated. If this energy has varied from the previous iteration beyond a predefined corridor, the entire PSDF is scaled to bring it back within limits. This step is crucial for controlling PGA drift and keeping the overall intensity of the earthquake under control.

6.5 Phase 2: Time-Domain Refinement

The most innovative feature of the program, in its 2025 version, is its second optimization phase, a high-precision process that is activated once the frequency-domain fitting has found the best candidate earthquake. This phase, managed by the `ADJUST_SIGNAL_IN_TIME_DOMAIN` subroutine, abandons the manipulation of the PSDF and works directly on the accelerogram’s time history to achieve a superior level of spectral compatibility.

6.5.1 The Finite State Machine: An Adaptive Optimization Strategy

The core of Phase 2 is a sophisticated **Finite State Machine (FSM)** that guides the adjustment process. Instead of applying a single correction strategy, the algorithm progresses through a series of up to five predefined optimization “states”. The process begins in State 1, with an aggressive strategy designed to correct large differences, and proceeds sequentially to more conservative states, culminating in State 5, which performs micro-adjustment polishing.

Each state is defined by a unique set of control parameters, including the initial strength of the corrections, their duration, and the rate at which failed attempts are attenuated. The program automatically transitions from one state to the next when it detects that the current strategy has stalled, i.e., when the improvement obtained in a full correction cycle falls below the `IMPROVE_THRS` threshold. This architecture allows the algorithm to adapt dynamically to the complexity of the problem, applying the most efficient strategy at each moment of the refinement. The user, in turn, can control the maximum number of optimization states to run (from 0 to 5) using the `MAX_STATE_TO_RUN` (`MaxStates`) parameter.

6.5.2 The Spectral Sweep Cycle

Within each state, the optimization is performed through iterative cycles called **spectral sweeps**. Each sweep consists of a complete pass through all periods of the spectrum and is composed of two fundamental steps:

1. Optimal Global Scaling. At the beginning of each sweep, an optimal scaling factor is calculated and applied to the entire accelerogram (subroutine `APPLY_OPTIMAL_SCALING`). This factor is derived analytically to minimize the least-squares difference between the calculated spectrum and the target spectrum. Its function is to efficiently adjust the overall energy of the signal, leaving the local corrections to handle the fine-tuning of the spectral shape.

2. Local Correction with Wavelets. Next, the algorithm iterates one by one through all the periods of the spectrum. For each period, it attempts to reduce the pointwise spectral difference by adding a small, time-localized wave, called a **corrective wavelet**. This local correction process is the heart of the refinement and is based on three key principles:

- **Wavelet Generation:** Each wavelet is constructed as a pure sinusoid whose frequency is exactly the inverse of the period to be corrected. This sinusoid is modulated by a **Hanning window**, a fundamental mathematical function in signal processing that ensures the wavelet starts and ends smoothly at zero, thus avoiding the introduction of discontinuities in the accelerogram [9]. This approach, which combines spectral and temporal analysis, is in turn inspired by concepts from wavelet analysis, a powerful tool for studying non-stationary signals [42]. The modulation is crucial to avoid introducing discontinuities and high-frequency noise into the accelerogram.
- **Intelligent Placement:** The effectiveness of a wavelet critically depends on where it is added to the time history. To minimize the disturbance of the PGA and other global characteristics of the signal, the wavelet is not applied at the instant of maximum spectral response (which often coincides with an energy peak in the accelerogram). Instead, the `FIND_OPTIMAL_WAVELET_CENTER` subroutine first identifies that ideal instant and then searches in its vicinity for a low-energy “valley”, i.e., a point of minimum absolute acceleration. Placing the wavelet at this low-amplitude point allows for a much more subtle, efficient, and less disruptive correction to the overall waveform.
- **Efficient Parabolic Search for the Optimal Scale Factor:** Determining the optimal “strength” or scale factor for each wavelet is an optimization problem in itself. To solve it efficiently, the program implements an adaptive search based on a parabolic fit. The process is as follows:
 1. **Initial Three-Point Sampling:** For each period, the algorithm evaluates the global equivalent spectral difference for three cases: without applying the wavelet (scale factor = 0), and applying it with predefined positive and negative scale factors.

2. **Parabolic Fit and Analytical Determination:** With these three points (scale factor, resulting error), the program fits a parabola and analytically determines the location of its vertex.
3. **Robust Decision Logic:** If the parabola is concave (with a well-defined minimum), the scale factor corresponding to the vertex becomes the candidate to be evaluated. If the parabola is convex or linear (without a clear minimum), the algorithm's robust logic identifies the direction of steepest descent and chooses a plausible factor at the boundary of the search space to avoid divergence.
4. **Final Evaluation and Selection:** The spectral difference for this fourth recommended scale factor is evaluated. Of the four evaluated cases (the initial three and the recommended one), the algorithm selects the factor that produced the lowest spectral difference.

The correction is only applied permanently if the best scale factor found is non-zero. If the best result is obtained with a factor of zero (i.e., by not applying the wavelet), the correction attempt is considered to have failed for that period, and the wavelet's "strength" is reduced for the next spectral sweep, allowing for increasingly fine adjustments. This approach significantly accelerates the optimization and improves precision by finding a near-optimal scale factor with only four evaluations per period.

6.5.3 Final PGA Scaling ("Windowed Scaling")

The `ADJUST_PEAK_GROUND_ACCELERATION` subroutine ensures that the final accelerogram rigorously respects the target PGA. The algorithm's behavior depends on whether the current PGA is higher or lower than the target:

- **If the current PGA exceeds the target:** The program employs an iterative "windowed scaling" method. The algorithm identifies the highest acceleration peak and applies a reduction factor to it. However, this factor is not applied uniformly but is modulated by a Hanning window centered on that peak. The result is a smooth, localized correction that reduces the peak to its target value but gradually fades to zero a short distance from it. The process is repeated, finding and correcting the new highest peak, until no point in the accelerogram exceeds the target.
- **If the current PGA is lower than the target:** The program performs a simple global scaling, multiplying the entire time history by the constant factor necessary for the highest peak to reach exactly the PGA value.

This dual approach ensures compliance with the PGA constraint in the least disruptive way possible. For excessive peaks, the windowed method minimizes the disturbance to the frequency content of the rest of the signal. For insufficient peaks, a simple linear scaling preserves the shape of the response spectrum, adjusting only its overall amplitude. In both cases, a final baseline correction is applied to ensure the kinematic consistency of the motion.

6.6 Fundamental Signal Processing Subroutines

Throughout the different phases of the algorithm, the program makes recurrent use of two signal processing techniques that are fundamental to ensuring the quality and physical realism of the resulting accelerogram.

6.6.1 Implicit Filtering by Frequency Band

Unlike programs that process real seismic records and require an explicit filtering step (e.g., with Butterworth or FFT filters), `earthquake_2025` does not need this step. Filtering is an **implicit and fundamental** feature of the synthesis method itself.

As described in Equation 6.1, the accelerogram is constructed as a sum of sinusoids. The frequency range of these sinusoids is strictly limited by the input parameters `T_CONT_MIN` (which defines the maximum frequency) and `T_CONT_MAX` (which defines the minimum frequency). Therefore, the program never generates energy outside this frequency band. This procedure acts, in practice, as a perfect band-pass filter, eliminating from the outset both the high-frequency noise and the very low-frequency drifts that plague real records and necessitate their filtering.

6.6.2 Baseline Correction (`CORRECT_BASELINE`)

For the velocity and displacement time series, obtained by integration, to be physically meaningful, it is imperative that they do not exhibit drifts at the end of the record. The `CORRECT_BASELINE` subroutine implements a robust analytical method that guarantees this condition. An additive correction to the acceleration is assumed to be linear: $a_{corr}(t) = C_1 + C_2 \cdot t$. The coefficients C_1 and C_2 are calculated by solving a 2x2 system of equations that mathematically imposes that the final velocity and displacement, obtained by integrating this correction, are exactly equal and opposite in sign to those of the uncorrected record. By subtracting this correction from the original accelerogram, it is ensured that the final result has zero velocity and displacement at the end time. This subroutine is called at multiple points in the algorithm (e.g., after applying the envelope and after each PGA adjustment) to maintain the kinematic integrity of the signal at all times.

Chapter 7

Methods of Analysis for the Seismic Response of Structures

7.1 Hierarchy and Selection of Analysis Methods

The fundamental objective of the seismic engineer is to calculate the response of a structure to a given seismic action to verify that it meets the pre-established performance objectives. Response is a general term that encompasses a set of critical parameters such as displacements, interstory drifts, floor accelerations, and internal forces like bending moments and shear forces. The selection of the analysis method to determine this response is not a trivial decision; it represents a fundamental balance between the accuracy required by the analysis, the expected physical behavior of the structure (linear or nonlinear), and the allowable computational effort.

No single method is optimal for all situations. Instead, the structural engineer has a range of analytical tools at their disposal, a set of methods that span from simplified and rapid procedures to high-fidelity numerical simulations. The mastery of the engineer lies in their ability to understand the hypotheses, advantages, and limitations of each of these methods, and in knowing how to choose the right tool for the problem at hand.

This chapter presents a technical description of the four most important seismic analysis methods used in professional practice and research. The mathematical equations and their descriptions are presented succinctly but rigorously, aiming for a dual objective: on one hand, to allow the experienced professional to quickly grasp the fundamental differences between each method; and on the other, to motivate the student or the engineer new to the subject to consult the bibliographic references for a more exhaustive study. Their theoretical foundations, mathematical formulations, and fields of application will be detailed, culminating in a comparative guide for selecting the most appropriate method.

1. Modal Response Spectrum Analysis (MRSA)
2. Modal Time-History Analysis (MTHA)
3. Implicit Direct-Integration Analysis (IDIA)
4. Explicit Direct-Integration Analysis (EDIA)

7.2 Method 1: Modal Response Spectrum Analysis (MRSA)

7.2.1 Theoretical Foundation and Procedure

Modal Response Spectrum Analysis (MRSA) is the reference method in most seismic design codes for structures with predominantly linear behavior. Its elegance and efficiency lie in two key concepts: **modal decomposition** and the use of the **design spectrum**.

The method is based on the principle of superposition, which is valid only for linear and elastically damped systems. The fundamental idea is that the response of a multi-degree-of-freedom (MDOF) system can be decomposed into the sum of the responses of a set of independent single-degree-of-freedom (SDOF) oscillators, where each SDOF represents a “mode” of vibration of the structure.

The procedure is as follows:

1. **Calculation of Dynamic Properties:** The eigenvalue problem for the undamped system is solved:

$$([K] - \omega_n^2[M])\{\phi_n\} = \{0\} \quad (7.1)$$

The solution of this equation provides the structure’s modes of vibration: the natural frequencies, ω_n , and the mode shapes, $\{\phi_n\}$.

2. **Calculation of Modal Participation:** For each mode n , the modal participation factor, Γ_n , is calculated, which quantifies the extent to which that mode is excited by the ground motion:

$$\Gamma_n = \frac{\{\phi_n\}^T[M]\{I\}}{\{\phi_n\}^T[M]\{\phi_n\}} \quad (7.2)$$

where $\{I\}$ is the influence vector of the ground motion (a vector of ones).

3. **Determination of Maximum Modal Response:** Herein lies the key to the method. Instead of solving for the response in time, the design spectrum is used directly. For each mode n , with its period $T_n = 2\pi/\omega_n$ and its damping ξ_n , the spectral displacement ordinate, $SD(T_n, \xi_n)$, is read from the spectrum. The maximum displacement response for that mode is calculated as:

$$\{u_{n,max}\} = \Gamma_n \{\phi_n\} SD(T_n, \xi_n) \quad (7.3)$$

4. **Statistical Combination of Modal Responses:** Since the maximums of each vibration mode generally occur at different instants in time, the total maximum response cannot be obtained by the direct sum of the modal maximums. Instead, it must be estimated using a statistical combination rule that approximates the most probable value of the maximum. The choice of the combination rule is not trivial and depends on the correlation between modal responses, primarily on the closeness of the structure’s natural frequencies. Table 7.1 summarizes the most important rules.

Table 7.1: Modal Combination Rules for the Estimation of Maximum Response.

Method	Formula for Maximum Response (R_{max})	Remarks and Applicability
ABSSUM (Absolute Sum)	$R_{max} \approx \sum_{n=1}^N R_{n,max} $	Obsolete and Overly Conservative. Assumes that all modal maximums occur at the same instant. It produces a theoretical upper bound but significantly overestimates the actual response. Its use in practice is not recommended.
SRSS (Square Root of the Sum of Squares)	$R_{max} \approx \sqrt{\sum_{n=1}^N (R_{n,max})^2}$	The Classic Method. Assumes that modal responses are statistically independent. It is very accurate for structures with well-separated natural frequencies (typically, $\omega_j/\omega_i > 1.15$). It may underestimate the response if there are modes with close frequencies.
CQC (Complete Quadratic Combination)	$R_{max} \approx \sqrt{\sum_{i=1}^N \sum_{j=1}^N \rho_{ij} R_{i,max} R_{j,max}}$ where ρ_{ij} is the modal correlation coefficient, which depends on the frequencies and damping of modes i and j .	The Current Standard. It is the most rigorous method and is recommended by most modern codes. It accounts for the correlation between vibration modes, providing accurate results even for structures with very close frequencies (torsionally coupled, etc.) [22].
10% Rule (Rosenblueth's Method)	SRSS is combined with an additional term for modes whose frequencies differ by less than 10%.	Obsolete. It was a simplified method to account for modal coupling before the CQC method became widespread. It has been completely superseded by the superiority and ease of implementation of CQC in modern software.

7.2.2 Scope of Application

MRSA is fast, efficient, and requires only the design spectrum, not a time history. It is the ideal method for the design and verification of the vast majority of conventional buildings and structures that are expected to remain essentially in the elastic range or exhibit very limited nonlinearities.

7.3 Method 2: Modal Time-History Analysis (MTHA)

Modal Time-History Analysis (MTHA) is a refinement of MRSA that, while still based on modal decomposition, calculates the evolution of the response over time.

7.3.1 Theoretical Foundation and Procedure

Steps 1 and 2 (calculation of modes and participation factors) are identical to those of MRSA. The fundamental difference lies in step 3:

3. **Solution of the Modal Equation in Time:** Instead of using the spectrum to obtain only the maximum value, the equation of motion for the modal coordinate $q_n(t)$ of each mode is solved over time, using a complete accelerogram, $a_g(t)$:

$$\ddot{q}_n(t) + 2\xi_n\omega_n\dot{q}_n(t) + \omega_n^2q_n(t) = -\Gamma_na_g(t) \quad (7.4)$$

This equation is solved numerically for each mode, obtaining the complete history $q_n(t)$.

4. **Superposition of Responses in Time:** The total physical response at any instant in time t is obtained by summing the contributions of each mode at that same instant:

$$\{u(t)\} = \sum_{n=1}^N \{\phi_n\}q_n(t) \quad (7.5)$$

7.3.2 Scope of Application

MTHA is still restricted to systems with **linear** behavior. Its main advantage over MRSA is that it provides the complete history of the response, which is useful in soil-structure interaction problems or for the design of vibration-sensitive equipment. Furthermore, by summing the responses at each instant, the approximation inherent in modal combination rules (SRSS, CQC) is avoided, resulting in a theoretically more exact solution (within the linear domain). Its major disadvantage is that it requires an excitation time history, either an accelerogram or a displacement history, which must be generated by programs like `earthquake_2025`.

7.4 Method 3: Implicit Direct-Integration Analysis (IDIA)

When the structure's behavior is nonlinear, modal decomposition loses its validity. Direct integration, a pillar of the Finite Element Method [63], emerges as the only rigorous path. In a method like Implicit Direct-Integration Analysis (IDIA), the complete, coupled system of equations of motion is solved.

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + \{F_R(u(t))\} = \{P(t)\} \quad (7.6)$$

where $\{F_R\}$ represents the internal resisting forces, which are now a nonlinear function of displacement.

7.4.1 The Implicit Method

In an **implicit** integration scheme, the solution at time $t + \Delta t$ is formulated as a function of the unknowns (displacements, velocities) at that same instant $t + \Delta t$. This leads to the need to solve a system of (generally nonlinear) equations at each time step.

One of the most widespread implicit methods is the family of **Newmark's methods**. These are based on the following assumptions for updating velocity and displacement:

$$\{\dot{u}\}_{t+\Delta t} = \{\dot{u}\}_t + [(1 - \gamma)\{\ddot{u}\}_t + \gamma\{\ddot{u}\}_{t+\Delta t}]\Delta t \quad (7.7)$$

$$\{u\}_{t+\Delta t} = \{u\}_t + \{\dot{u}\}_t\Delta t + [(\frac{1}{2} - \beta)\{\ddot{u}\}_t + \beta\{\ddot{u}\}_{t+\Delta t}]\Delta t^2 \quad (7.8)$$

where β and γ are parameters that control the accuracy and stability of the method. By combining these equations with the dynamic equilibrium equation at $t + \Delta t$, and after a linearization process (such as Newton-Raphson), one arrives at an incremental system of equations of the form [43]:

$$[K_{eff}]\{\Delta u\} = \{F_{eff}\} \quad (7.9)$$

where $[K_{eff}]$ is the effective stiffness matrix, which depends on the system's tangent stiffness matrix $[K_T]$. The need to assemble and solve this system of equations in each iteration is the distinguishing feature of the method.

7.4.2 Scope of Application

Implicit methods are the tool of choice for the vast majority of nonlinear structural dynamics problems, including seismic analysis. For linear systems, they are unconditionally stable if one chooses $\gamma \geq 0.5$ and $\beta \geq 0.25(0.5 + \gamma)^2$, which allows for the use of large time steps. It is important to note that if $\gamma = 0.5$ is selected, the method conserves energy, whereas if $\gamma > 0.5$, a **numerical damping** (or algorithmic damping) is introduced that filters out spurious high frequencies, a highly desirable property in complex models to facilitate convergence. Its robustness makes it the method of choice in general-purpose finite element programs like Abaqus/Standard [1]. Its load input is a time history, typically of prescribed displacements at the base (though an acceleration history can also be used), which must be generated by programs like `earthquake_2025`.

7.5 Method 4: Explicit Direct-Integration Analysis (EDIA)

In an **explicit** scheme, such as Explicit Direct-Integration Analysis (EDIA), the solution at time $t + \Delta t$ is calculated using only known values from previous steps (typically t and $t - \Delta t$). The great advantage is that it does not require solving a system of equations. The equation of motion is used to directly “solve for” the acceleration at time t :

$$\{\ddot{u}\}_t = [M]^{-1}(\{P\}_t - \{F_{int}\}_t) \quad (7.10)$$

where $\{F_{int}\}_t$ are the internal forces. If the mass matrix $[M]$ is diagonal (which is easily achieved with a lumped mass formulation), its inversion is trivial.

7.5.1 The Explicit Method

The **central difference** method is the most common explicit scheme. Velocity and acceleration at time t are approximated as:

$$\{\dot{u}\}_t \approx \frac{1}{2\Delta t}(\{u\}_{t+\Delta t} - \{u\}_{t-\Delta t}) \quad (7.11)$$

$$\{\ddot{u}\}_t \approx \frac{1}{\Delta t^2}(\{u\}_{t+\Delta t} - 2\{u\}_t + \{u\}_{t-\Delta t}) \quad (7.12)$$

By substituting the second into the equation of motion and rearranging, $\{u\}_{t+\Delta t}$ can be directly solved for without solving any matrix system.

7.5.2 Scope of Application and Conditional Stability

The per-step computational efficiency of explicit methods is very high. However, their main disadvantage is that they are **conditionally stable**. The theory of numerical methods shows that the solution diverges explosively if the time step, Δt , is not smaller than a critical value, Δt_{crit} . This critical value is governed by the period of the highest vibration mode of the meshed system, T_{min} [7]:

$$\Delta t \leq \Delta t_{crit} = \frac{T_{min}}{\pi} \quad (7.13)$$

Since T_{min} can be extremely small in fine meshes, explicit methods require a very large number of time steps. They are, therefore, ideal for events where the nature of the problem already demands the use of very small time increments (such as in earthquake simulation to capture high frequencies, or in impact and explosion problems). Their efficiency also stands out in problems with very complex nonlinearities (contact, extreme damage) where the convergence of an implicit method can be difficult.

7.6 Criteria for Selecting the Analysis Method

The fundamental premise that should guide the engineer is: *There is no universal method; each method is a tool with a purpose, advantages, and limitations.*

7.6.1 Modal Response Spectrum Analysis (MRSA)

- **Key Advantages:** Very fast and computationally efficient. Only requires the design spectrum. It is the standard method in design codes.
- **Disadvantages and Limitations:** Strictly applicable only to linear systems. Does not provide the time history of the response. Modal combination is a statistical approximation.
- **Recommended Use Case:** Preliminary and final design of conventional structures with elastic behavior. Quick verification according to codes.
- **Required Knowledge Level:** Basic / Intermediate.

7.6.2 Modal Time-History Analysis (MTHA)

- **Key Advantages:** Provides the complete time history. It is more accurate than MRSA for linear systems. Computationally faster than direct integration.
- **Disadvantages and Limitations:** Strictly limited to linear systems. Requires a complete time history of the excitation (accelerations or displacements).
- **Recommended Use Case:** Analysis of linear structures where the time evolution is needed (e.g., interaction with equipment, P-Delta effects).
- **Required Knowledge Level:** Intermediate.

7.6.3 Implicit Direct-Integration Analysis (IDIA)

- **Key Advantages:** It is the most robust method for nonlinear analysis. Unconditionally stable for linear systems.
- **Disadvantages and Limitations:** Computationally expensive (solves a system of equations at each step). Convergence can be difficult in severe nonlinearities. Requires a time history.
- **Recommended Use Case:** The method of choice for Performance-Based Design (PBD) and the nonlinear seismic analysis of critical structures. The industry standard.
- **Required Knowledge Level:** Advanced / Expert.

7.6.4 Explicit Direct-Integration Analysis (EDIA)

- **Key Advantages:** Very efficient per time step. Excellently handles extreme nonlinearities (contact, collapse).
- **Disadvantages and Limitations:** Conditionally stable. The need to use a very small time step (limited by the smallest element in the mesh) can make it computationally intensive for long-duration events, compared to an implicit analysis where large steps can be used.
- **Recommended Use Case:** Analysis of problems with very severe nonlinearities (contact, collapse) or when the model's accuracy requires an intrinsically small Δt . For seismic analysis, it is a robust alternative to the implicit method, especially in very large or complex models where the convergence of the implicit analysis can be a challenge. Its efficiency becomes competitive if the Δt required for accuracy is up to 100 times the stability Δt_{crit} [8, 32].
- **Required Knowledge Level:** Advanced / Expert.

7.7 The Indispensable Relevance of Signal Generation

This review of analysis methods leads to a fundamental conclusion: *the most advanced and accurate methods (MTHA, IDIA, and EDIA), which are indispensable for addressing nonlinearity and obtaining a complete description of the response, inescapably require a **complete and high-quality time history of the seismic excitation**.*

Herein lies the critical importance of the `earthquake_2025` program. It acts as the indispensable connecting piece that translates the seismic hazard, defined by the code in the form of an elastic design spectrum, into a time-domain signal that serves as *input* for these advanced analyses (MTHA, IDIA, or EDIA). The program's new hybrid methodology goes a step further: by not only generating a statistically compatible signal but also refining it to achieve "super-compatibility" with the elastic spectrum, it ensures that the *input* for the structural analysis contains the most accurate possible damage potential and energy demand. This is especially critical for Performance-Based Design and for the analysis of nonlinear systems, where the response can be highly sensitive to the characteristics of the excitation.

7.8 Application of the Seismic Excitation

Once a high-quality time history has been generated, the next crucial step is its correct implementation in the numerical model. The manner in which the seismic excitation is applied depends fundamentally on the scope and complexity of the model, with the main distinction being between fixed-base models and models that include Soil-Structure Interaction (SSI) (*Soil-Structure Interaction*).

7.8.1 The Recommended Approach: Displacement Excitation at the Base

For direct integration methods (IDIA and EDIA), it is often more numerically stable, accurate, and conceptually more correct to apply the excitation as a prescribed displacement at the base nodes (a **displacement history**) rather than an acceleration history. The reason is that structures naturally respond to relative movements, and prescribing a displacement avoids the accumulation of integration errors that can occur when applying accelerations. The `earthquake_2025` program, by generating a high-quality acceleration history with proper baseline correction, obtains through double integration a robust and physically consistent **displacement history**, which it provides ready to be used in the most sophisticated simulations.

7.8.2 Basal Excitation in Advanced Models

The recommendation to apply the seismic excitation as a displacement history at the base of the superstructure, as has been argued in Section 7.8.1, constitutes a robust and widely accepted practice in the field of structural analysis. This approach, which conceptually simulates the imposition of the free-field ground motion onto the foundation, is the cornerstone of non-linear time-history analyses (NLTHA) for the vast majority of conventional buildings and structures.

However, at the forefront of seismic engineering, there are more complex modeling scenarios whose correct implementation demands a deeper understanding of the nature of seismic excitation. It is imperative for the specialist engineer to recognize the context in which this standard practice is valid and when more advanced formulations must be employed, particularly in the field of **Soil-Structure Interaction (SSI)**.

In an SSI analysis, the objective transcends the response of the superstructure; it seeks to capture the coupled behavior of the complete system, including a significant volume of the foundation soil and, at times, an adjacent fluid (as in the case of dams and marine structures). In this context, the model is no longer excited at the base of the structure, but at the boundaries of this extended soil domain [26, 27, 13]. This distinction is fundamental and gives rise to two critical considerations:

1. **The Need for Absorbent Boundaries:** When modeling a subdomain of a theoretically semi-infinite medium like the soil, it is crucial to avoid the spurious reflection of waves at the artificial boundaries of the model. The waves generated by the seismic excitation and, of equal importance, the waves radiated by the structure itself as it vibrates and the reflection at the ground surface, must be able to exit the model domain without returning as a non-physical echo.

The inability to allow this energy radiation eliminates a fundamental damping mechanism—**radiation damping**—which can lead to a significant overestimation of the structural response. The standard solution to this problem is the use of **infinite elements** or viscous absorbing boundaries at the edges of the soil domain, thus simulating the condition of an semi-infinite elastic space.

2. **The Nature of the Seismic Load:** Applying the load at these absorbent boundaries introduces a conceptual paradox. If a motion (displacement or acceleration) is prescribed at a boundary, it becomes, by definition, a non-free wall that can reflect the waves arriving from the structure.

The most physically pure formulation to avoid this contradiction requires a different approach. In this rigorous method, the basal boundary is left free, and the seismic excitation is introduced as a **shear stress load** representing the incident wave traveling only upwards from the bedrock.

The conversion of a seismic motion record to a stress history is based on the impedance relationship for a plane shear wave. The shear stress, $\tau(t)$, to be applied at the basal boundary is directly proportional to the particle velocity of the incident wave, $v_{inc}(t)$:

$$\tau(t) = (\rho V_S) \cdot v_{inc}(t) \quad (7.14)$$

where ρ is the density of the medium and $V_S = \sqrt{G/\rho}$ is the shear wave velocity, with G being the shear modulus. The term ρV_S (equivalent to $\sqrt{G\rho}$) is known as the impedance of the medium, whose units are Mass/(Area · Time) [e.g., kg/(m² · s)].

To determine the correct magnitude of this load, it is necessary to combine two fundamental physical phenomena that act in opposite directions regarding the signal amplitude.

First, the absorbent boundary (the infinite element) behaves mechanically as a viscous dashpot. To successfully introduce a wave into the numerical domain through this mechanism, the applied force must be **twice** the force associated with the incident wave ($2 \cdot \tau_{inc}$); this is because half of the applied energy is dissipated in overcoming the resistance of the viscous dashpot itself, allowing the other half to effectively enter the model as kinetic and strain energy.

Second, the origin of the seismic signal must be considered. The record $v_g(t)$ provided by the `earthquake_2025` program corresponds to the motion at the free **surface** of the ground. Due to the phenomenon of total reflection at the free surface, the amplitude of the recorded motion is twice the amplitude of the incident wave traveling from the bedrock ($v_g = 2 \cdot v_{inc}$). Consequently, the velocity of the incident wave that we must theoretically inject at the base is half of the surface record: $v_{inc}(t) = 0.5 \cdot v_g(t)$.

Fortunately, these two effects mathematically cancel each other out exactly: the doubling factor required by the absorbent boundary ($\times 2$) is compensated by the reduction factor needed to obtain the incident wave from the surface record ($\times 0.5$). As a result, the shear stress history, $\tau(t)$, to be applied at the base of the model is calculated directly using the impedance of the medium without additional factors:

$$\tau(t) = \sqrt{G\rho} \cdot v_g(t) \quad (7.15)$$

This history of $\tau(t)$ is introduced as a distributed **shear load** at the nodes of the interface between the finite elements and the absorbent infinite elements. It is crucial to emphasize that no kinematic constraint (displacement) is imposed on these nodes;

they remain free, allowing the waves reflected by the structure that return to the base to pass through the boundary and be correctly absorbed by the infinite elements, emulating the radiation of energy into the infinite half-space.

Convolution, Deconvolution, and Foundation Input Motion Processes. It is imperative to make a conceptual and practical clarification regarding spectral consistency when modeling the foundation soil:

- **The Direct Process (Convolution):** From the perspective of one-dimensional shear wave propagation, the transfer of seismic motion from the rigid bedrock (*rock outcrop*) to the free surface is modeled as a mathematical convolution between the incoming basal motion and the complex transfer function of the soil column. The stratigraphy acts as a dynamic filter that amplifies specific frequency bands and attenuates others due to material damping.
- **The Inverse Problem (Deconvolution):** When the engineer has a spectrum or record defined at the surface of a soft soil deposit and needs to determine the coherent excitation at the underlying rigid base to calibrate a numerical model, the inverse process or **deconvolution** is required. Through this procedure, the surface signal is “deconvolved” using the dynamic properties of the strata to deduce the motion that must have occurred at depth in the bedrock.

Implications for the use of earthquake₂₀₂₅: The program generates accelerograms directly compatible with the target elastic spectrum provided by the user. In standard engineering practice, this target spectrum represents the code demand at the ground surface (i.e., it already accounts for soil amplification factors such as S , F_a , or F_v). Consequently:

- a) In a **fixed-base model** (superstructure alone), the generated time history represents the free-field surface motion and is prescribed directly at the foundation supports.
- b) In an **SSI model with an extended soil domain**, directly injecting the surface motion at the bottom of the numerical domain would constitute a serious conceptual error if stratigraphic contrasts or property variations with depth exist, as the model’s soil column would amplify the signal again, leading to a spurious “*double amplification*”. In such heterogeneous cases, it is mandatory to first apply a numerical deconvolution to the generated signal (using equivalent-linear formulations or frequency-domain methods; see, e.g., [50, 55, 62]) to obtain the equivalent basal wave train that, after propagating through the finite element mesh, faithfully reproduces the target design spectrum at the surface. However, if the modeled soil layer is completely homogeneous and uniform (without impedance contrasts or significant material damping along the path), the signal undergoes no significant dynamic alterations during its vertical propagation, and therefore it can be introduced directly at the bottom boundary without modification.

In conclusion, while the methodology of applying a displacement history at the base of the structure is the fundamental and correct tool for time-domain structural analysis, the

engineer must be aware that when extending the scope of the model to explicitly include the soil domain, the rules for applying the load and treating the boundaries evolve.

The understanding of these advanced concepts of Soil-Structure Interaction is what distinguishes the expert analyst and ensures the highest fidelity in the most complex and critical simulations.

Chapter 8

Seismic Protection Systems and Their Analysis Requirements

8.1 Beyond Strength and Ductility

The traditional philosophy of earthquake-resistant design, described in previous chapters, is based on providing the structure with an adequate combination of stiffness, strength, and, crucially, ductility. In this strategy, the energy imparted by the earthquake is dissipated internally through inelastic deformation and controlled damage to the main structural elements (beams, columns, walls). While this is a robust and proven approach to ensure life safety, it has an unavoidable consequence: the structure may suffer significant damage and require costly repairs or even demolition after a severe earthquake.

Modern seismic engineering, in its pursuit of a higher level of performance and resilience, has developed an alternative philosophy: instead of designing the structure to “resist” the earthquake through damage, the goal is to modify the overall seismic response by incorporating specialized devices. These **seismic protection systems**, which are part of the broader field of structural control [57], act intelligently to reduce the demand imposed on the structural system or to manage energy dissipation more efficiently.

This chapter explores the three major strategies of passive seismic protection, their physical mechanisms, and, fundamentally, the analysis requirements they impose, demonstrating why time-domain analysis is an indispensable tool for their correct design and verification.

8.2 Strategy 1: Seismic Base Isolation Systems

8.2.1 Fundamental Principle

Seismic base isolation is perhaps the most intuitive and radically effective seismic protection strategy. Its fundamental principle consists of **decoupling** the superstructure from the ground motion. This is achieved by introducing an interface of very high horizontal flexibility between the foundation and the base of the structure.

By significantly lengthening the structure's first fundamental period of vibration (shifting it, for example, from 0.5 to 2.9 seconds), the “isolated” system moves into the long-period, low-acceleration region of the response spectrum (see Figure 8.1). The consequence is a drastic reduction in the inertial forces and accelerations transmitted to the superstructure, which, ideally, can remain practically in the elastic range even during a severe earthquake. The seismic motion is concentrated in large horizontal displacements within the isolation system, which is specifically designed to accommodate them.

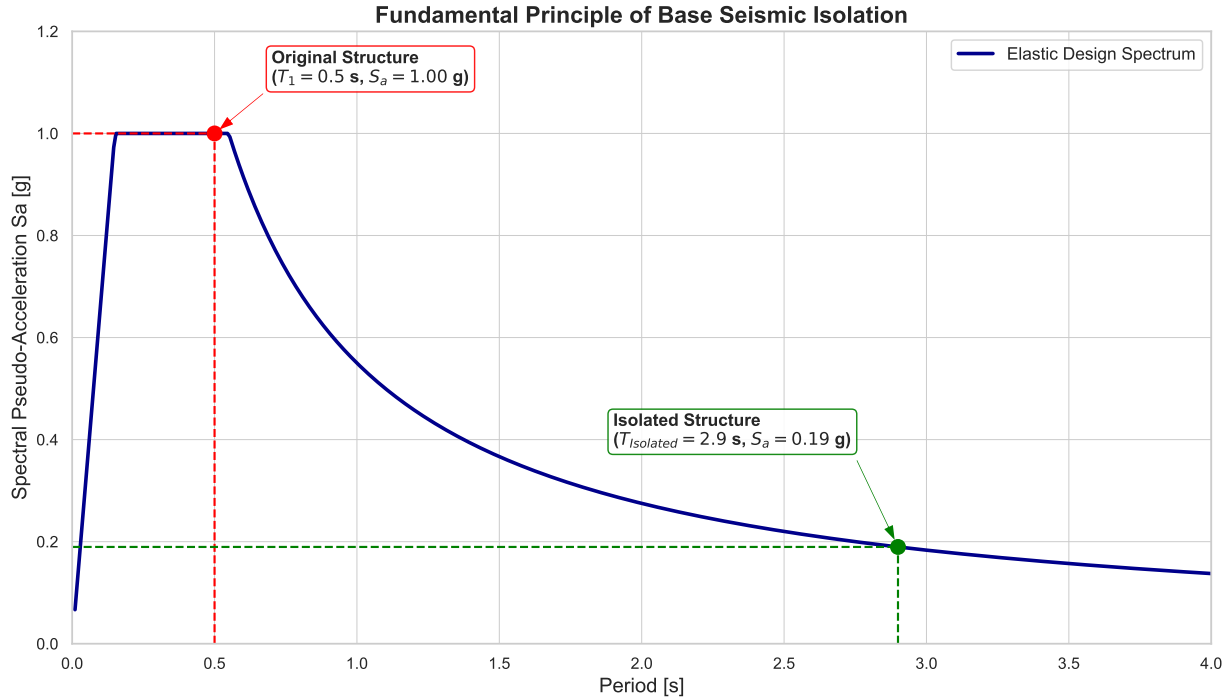


Figure 8.1: Principle of seismic isolation: By lengthening the fundamental period from T_1 to $T_{isolated}$, the spectral acceleration demand S_a is drastically reduced.

8.2.2 Types of Devices

- Elastomeric Bearings:** These consist of alternating layers of elastomer (natural rubber or neoprene) and thin vulcanized steel plates. The elastomer provides horizontal flexibility, while the steel plates provide high vertical stiffness to support gravity loads. There are two main variants: **HDR** (High Damping Rubber Bearings), which use a rubber compound with additives to increase energy dissipation, and **LRB** (Lead Rubber Bearings), which introduce a central lead core (see Figure 8.2). This core yields plastically under seismic motion, providing a very stable and reliable hysteretic energy dissipation capacity that limits excessive displacements [56, 37].

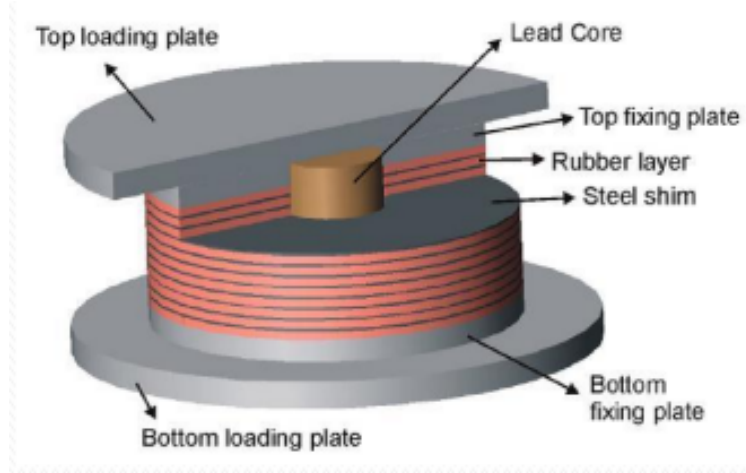


Figure 8.2: Schematic of a Lead Rubber Bearing (LRB), showing the layers of rubber and steel, and the central dissipating core.

- **Friction Pendulum Systems (FPS):** The FPS consists of a concave stainless steel surface on which an articulated slider coated with a low-friction material moves. The curvature of the surface defines the effective period of the system ($T = 2\pi\sqrt{R/g}$, where R is the radius of curvature), while the friction between the slider and the surface provides energy dissipation (see Figure 8.3). Its behavior is highly nonlinear [17].

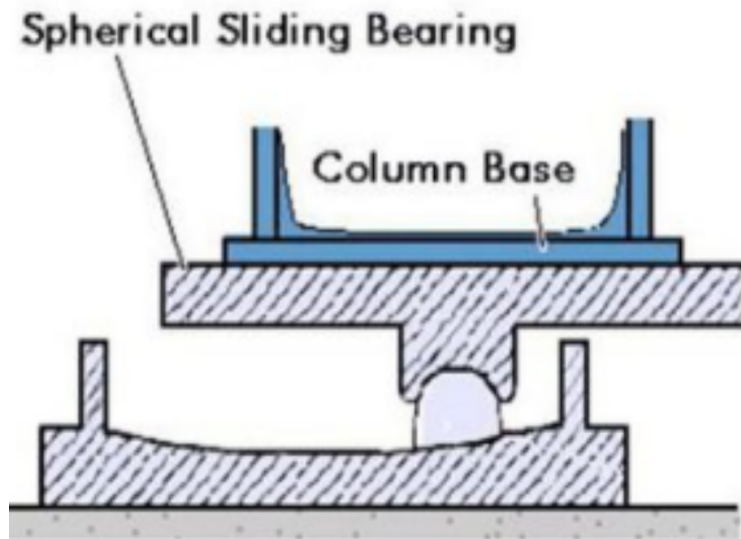


Figure 8.3: Operating principle of a Friction Pendulum System (FPS), where sliding on a concave surface generates the restoring force.

8.2.3 Analysis Requirements

The nature of the behavior of isolation systems imposes very strict analysis requirements:

- **High Nonlinearity:** The force-displacement behavior of isolators is intrinsically non-linear (bilinear for LRB, frictional for FPS). This nonlinearity is essential for their function and cannot be adequately approximated by linear methods.
- **Sensitivity to Time History:** The response of isolators, especially maximum and residual displacements, is highly sensitive to the characteristics of the complete accelerogram, particularly to long-period velocity pulses (“fling-step” or directivity effects) that cannot be represented by a response spectrum.

For these reasons, all modern seismic codes, such as ASCE 7, **mandatorily require** the use of Non-Linear Time History Analysis (NLTHA) for the design of base-isolated structures. Since the response of these systems, and particularly their large displacements, is extremely sensitive to the characteristics of the excitation, the use of artificial earthquakes with a very high degree of spectral compatibility, such as those generated by `earthquake_2025`, becomes an indispensable requirement for obtaining reliable results.

8.3 Strategy 2: Supplemental Energy Dissipation Systems

8.3.1 Fundamental Principle

The objective of this strategy is to increase the damping capacity of the overall structural system. Instead of allowing seismic energy to be dissipated through damage to beams and columns, devices (commonly called “dampers”) are installed that are specifically designed to absorb most of this energy and dissipate it as heat. This protects the main structure, reduces displacements and accelerations, and allows it to remain elastic or with minimal damage. The dampers act as structural “fuses” that can be inspected and replaced after an earthquake.

8.3.2 Types of Devices

Dampers are generally classified according to the physical mechanism they depend on:

- **Velocity-Dependent:** The most common example is **viscous fluid dampers**. They generate a force that is proportional to the relative velocity between their ends, $F = C \cdot v^\alpha$. If $\alpha = 1$, the damper is linear; if $\alpha < 1$, it is nonlinear.
- **Displacement-Dependent:** These devices generate forces that depend on relative displacement.
 - **Friction Dampers:** They dissipate energy through the friction generated when two surfaces slide against each other after overcoming a pretensioning force.

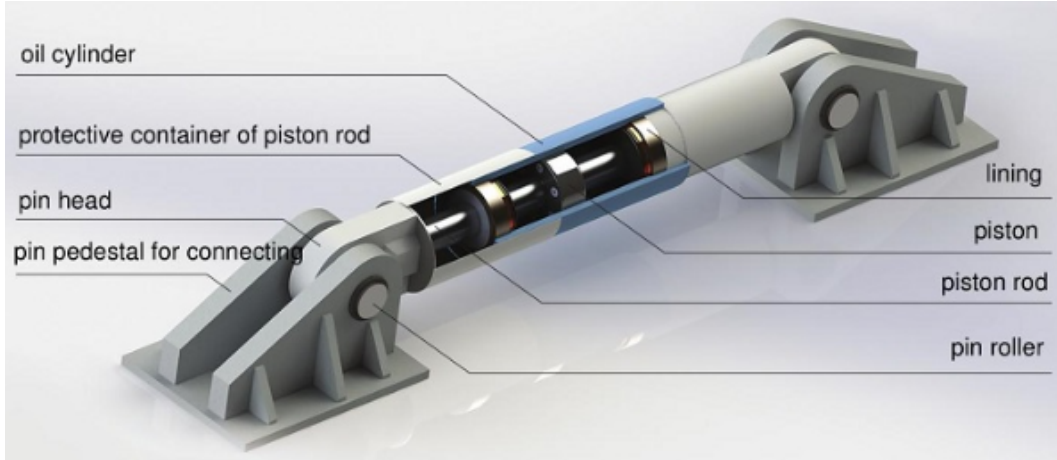


Figure 8.4: Cross-section of a viscous fluid damper, showing the piston moving through a silicone oil to generate the damping force.

- **Yielding (or Hysteretic) Dampers:** They use the plastic deformation of metals to dissipate energy. Examples are the **ADAS** (Added Damping and Stiffness) (see Figure 8.5) and **TADAS** (Triangular Added Damping and Stiffness) devices.



Figure 8.5: Example of an ADAS hysteretic (or yielding) energy dissipator. Seismic energy is dissipated through the plastic deformation of the curved metal plates.

8.3.3 Analysis Requirements

For structures with *linear* viscous dampers, some codes allow the use of modified spectral methods that use an “effective damping” greater than 5%. However, this is an approximate approach. For a rigorous design and for all cases with nonlinear dampers (which are the vast majority), **Non-Linear Time History Analysis (NLTHA) is the only method** that can correctly capture the cycle-by-cycle interaction between the structure and the devices.

The accuracy of this analysis depends critically on the quality of the input signal, so the use of time histories with superior spectral compatibility, like those generated by `earthquake_2025`, is fundamental for a reliable performance assessment.

8.4 Strategy 3: Tuned Mass Damper (TMD) Systems

8.4.1 Fundamental Principle

A Tuned Mass Damper (TMD) is a passive control system consisting of a secondary mass-spring-damper system that is added to the main structure. The key to its operation is **tuning**: the natural frequency of the TMD is adjusted to be very close to the fundamental frequency of the structure to be controlled.

When the structure begins to vibrate at its fundamental frequency due to an external excitation, the TMD enters into resonance and begins to oscillate with a large amplitude, but with a phase lag of approximately 90 degrees with respect to the structure. The motion of the TMD's mass generates an inertial force that opposes the motion of the main structure, transferring vibrational energy from the structure to the TMD, where it is dissipated by the device's damper. The most famous example is the 660-ton pendulum suspended at the top of the Taipei 101 skyscraper (see Figures 8.6 and 8.7).

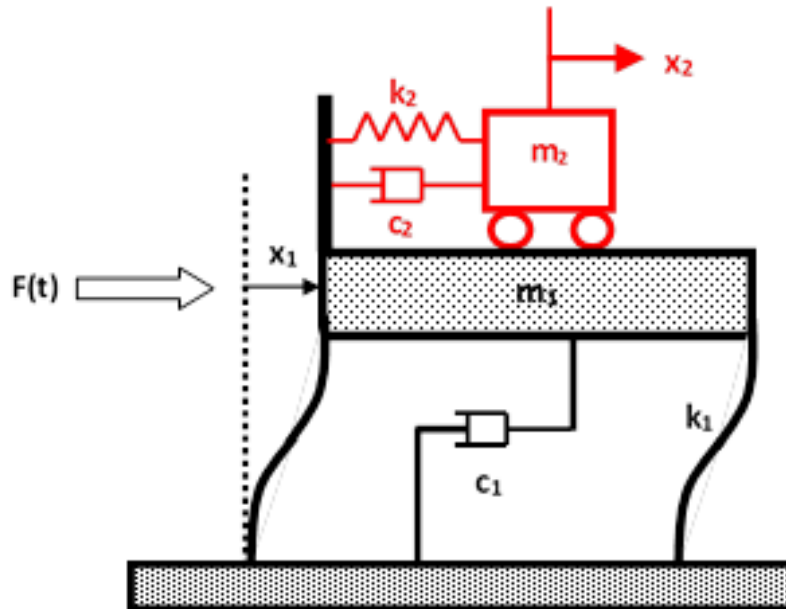


Figure 8.6: Schematic diagram of the operating principle of a Tuned Mass Damper (TMD). The secondary system (m_2, k_2, c_2) is added to the main structure (m_1, k_1, c_1) to control its response.

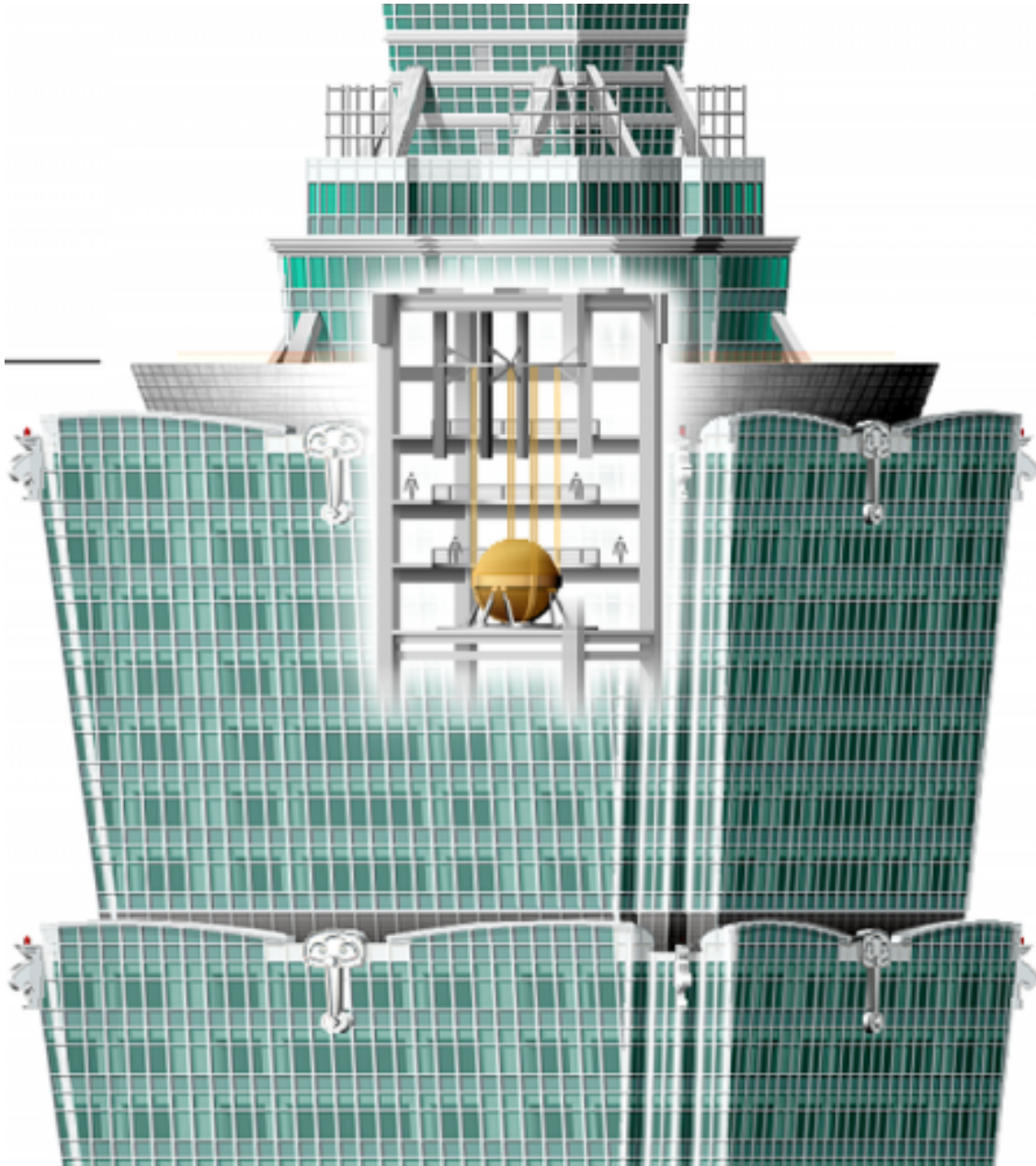


Figure 8.7: Example application of a TMD: the 660-ton pendulum of the Taipei 101 skyscraper, designed to counteract vibrations from wind and earthquakes.

8.4.2 Analysis Requirements

The analysis of structures with TMDs depends on the expected behavior of the main structure:

- If the main structure is designed to remain **elastic**, the interaction with the TMD (which is also a linear system) can be accurately modeled using Modal Response Spectrum Analysis (MRSA) or Modal Time-History Analysis (MTHA) on the combined system.
- However, if the main structure is expected to enter the **nonlinear range**, its dynamic properties (stiffness and period) change over time. This causes a “detuning” phenomenon of the TMD, leading to a loss of its effectiveness. The complex interaction between a nonlinear primary system and a linear secondary system can only be reliably solved through a **Non-Linear Time History Analysis (NLTHA)**. For this analysis to correctly capture the detuning phenomenon, it is crucial that the excitation used (the time history) represents the energy in each frequency band with maximum fidelity, a condition that the “super-compatible” signals generated by `earthquake_2025` exceptionally fulfill.

8.5 Conclusion: The Unavoidable Need for Time-History Analysis

The analysis of seismic protection systems leads to a unanimous and compelling conclusion: *their rigorous design and verification require, in the vast majority of cases, the execution of nonlinear time-domain analyses.* The linearity assumptions on which simplified spectral methods are based are generally invalidated by the very nature of the behavior of these advanced devices.

Therefore, Non-Linear Time History Analysis (**NLTHA**) is not merely an academic option but a fundamental requirement of state-of-the-art engineering practice. This reality, in turn, creates an indispensable demand for high-quality time histories that are realistic and compatible with the design seismic hazard for a specific site, underscoring the central role and critical importance of simulation tools. The `earthquake_2025` program, with its ability to generate “super-compatible” signals, acts as the indispensable bridge that connects the code’s definition of seismic hazard, given in spectra, with the reliable execution of the most sophisticated analysis methods that modern engineering demands.

Chapter 9

Conclusions and Future Lines of Research

9.1 Summary and Synthesis of the Monograph

This monograph has undertaken a comprehensive journey through the concepts and tools necessary for the simulation of seismic ground motions for their application in structural analysis. The text has been structured into two complementary parts, with the intention of serving both as a theoretical reference document and a practical guide for a specific computational tool.

Part I laid the theoretical foundations, exploring the physics of the seismic phenomenon, from the tectonic origin of earthquakes and wave propagation to the characterization of their effects at a site. Fundamental concepts such as magnitude and intensity have been defined and differentiated, and the tools used by engineers to quantify seismic demand have been analyzed in depth: the accelerogram, response spectra, and integral measures of energy, such as Arias Intensity. Likewise, the need for these tools has been contextualized within the philosophy of modern earthquake-resistant design, particularly Performance-Based Design, which requires the use of nonlinear time-history analysis.

Part II, in turn, has presented the materialization of this theory into a robust computational tool: the `earthquake_2025` program. The internal algorithmic methodology has been detailed, from the generation of the initial stochastic signal to the refined iterative process that ensures the compatibility of the generated accelerograms with a target spectrum. Finally, a detailed guide for the use of the program and the critical interpretation of its numerical and graphical results has been provided, thus connecting practical application with the previously established theoretical foundations. The common thread of the work has, therefore, been this journey from the “why” of physics and engineering to the “how” of computational implementation.

9.2 Capabilities and Limitations of the earthquake_2025 Program

9.2.1 Main Capabilities of the 2025 Version

The `earthquake_2025` program introduces significant advancements that establish it as a new-generation tool for seismic simulation. Its key strengths are:

- **Two-Phase Hybrid Methodology:** The most outstanding capability is its new optimization algorithm. It combines a robust adjustment in the frequency domain (Phase 1) with a novel and powerful refinement in the time domain using corrective *wavelets* (Phase 2).
- **Superior Spectral Compatibility:** Thanks to the second optimization phase, the program is capable of generating signals with an extraordinarily high level of fit to the target spectrum, minimizing residual discrepancies and producing “super-compatible” accelerograms.
- **Robust Convergence Algorithms:** Both Phase 1 (with its smoothed and energy-controlled PSDF correction) and Phase 2 (with its intelligent wavelet placement and adaptive strength) are designed to ensure stable and efficient convergence.
- **Clear and Concise Results Output:** The structure of the output file has been redesigned to be more intuitive, clearly presenting the contribution of each optimization phase and a complete final summary with all relevant engineering metrics (PGA, PGV, PGD, Arias Intensity, etc.).
- **Transparency and Accessibility:** Like its predecessors, the program maintains its open-source philosophy (MIT License) and its implementation in Fortran 77, ensuring the verifiability of the code and its compatibility with the ecosystem of computational engineering and academic research.

9.2.2 Current Limitations and Scope of Application

Like any numerical model, `earthquake_2025`, in its current version, is based on a set of hypotheses that define its scope of application and its limitations. It is crucial that the user is aware of the following limitations:

- **Stationarity of Frequency Content:** The underlying model for signal generation in Phase 1 is based on a Power Spectral Density Function (PSDF) that is constant in time. Non-stationarity in amplitude is introduced through the envelope function. However, the algorithm’s Phase 2, by adding corrective *wavelets* at specific time instants, de facto introduces a slight non-stationarity in the frequency content of the final signal. Despite this improvement, the model is not explicitly designed to simulate the time evolution of frequencies observed in real earthquakes (e.g., the sequential arrival of P, S, and surface waves). A rigorous treatment of this phenomenon would require the use

of evolutionary power spectral densities (EPSD), such as those proposed by Shinozuka and Sato [54].

- **Absence of Spatial Variability:** The program generates a single time history, assuming that the ground motion is identical at all points of the structure's base. This hypothesis is valid for structures with compact foundations. However, for spatially extended structures (long-span bridges, viaducts, dams, pipelines), the spatial variability of seismic motion (loss of coherence, wave-passage effect) can have a significant impact on the response, a phenomenon not captured by the current model. The importance of spatial variability was highlighted in pioneering works such as those by Wolf J.P. [61].

9.3 Main Contributions of the Work

It is considered that the main contribution of this work is twofold. On one hand, the provision of the `earthquake_2025` program to the academic and professional community as an open-source, robust, validated, and documented tool, which allows for rigorously addressing the nonlinear time-history analyses required by modern seismic engineering.

On the other hand, this monograph itself aims to be a contribution. By unifying in a single text the theoretical foundations of seismology and structural design with the detailed description of a computational methodology and a practical application guide, it is hoped to offer a self-contained resource that is useful both for the training of new researchers and postgraduate students and for consultation and application by practicing engineers.

9.4 Future Lines of Research and Development

The field of seismic ground motion simulation is an active area of research, and the `earthquake_2025` program can serve as an excellent platform for future extensions. Some possible lines of development include:

- **Implementation of Non-Stationary Frequency Models:** Overcoming the current limitation by incorporating models based on evolutionary power spectral densities (EPSD), which allow the PSDF to vary over time. This approach would permit a more realistic simulation of the sequential arrival of different wave types. Various formulations exist to address this problem, such as those based on the discretization of the Fourier amplitude spectrum [47] or those using wavelet theory [42].
- **Generation of Spatially Variable Seismic Fields:** Extending the methodology to generate multiple correlated accelerograms at different points in space. This would involve implementing a spatial coherence model, enabling the seismic analysis of extended structures [10].
- **Integration with Structural Analysis Software:** Developing interfaces or modules that allow direct communication between `earthquake_2025` and finite element analysis programs, streamlining the nonlinear analysis workflow.

- **Development of a Graphical User Interface (GUI):** To facilitate its use by a wider audience and in educational settings, the creation of a GUI that simplifies the creation of input files and the visualization of results would be a valuable step.

9.5 Author's Final Words

The journey that began more than three decades ago with doctoral research has culminated in the tool and the monograph that the reader holds in their hands. Seismic engineering is a constantly evolving discipline, where collaboration between academic research and professional practice is fundamental. It is my sincerest wish that the `earthquake_2025` program and this text serve as a bridge between both worlds, and that they become useful tools for building a safer and more resilient environment against the seismic threat. The decision to offer both resources openly responds to a firm commitment to the collective advancement of knowledge in our profession.

Appendices

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Appendix A

Dictionary of Input and Output Parameters

This appendix serves as a quick and detailed reference for all parameters used by the `earthquake_2025` program. It is divided into two sections: the first describes the parameters that the user must provide in the input file, while the second details the values found in the output file.

A.1 Input File Parameters

The following are all the variables read from the input file (Unit 5), organized by thematic blocks according to their reading order.

A.1.1 Block 1: Title and Units

Parameter	Description and Details
TITLE	Description: Main title of the simulation (up to 80 characters). It is printed in the output header. Type: CHARACTER*80
DIST_UNIT_RAW	Description: Base distance unit text, used only for labeling outputs (e.g., [m], [cm], [in], [ft]). Type: CHARACTER*80

A.1.2 Block 2: Definition of Periods and Frequencies

Parameter	Description and Details
T_SPEC_MIN	Description: Lower limit (shortest period) for the definition of the response spectrum. Type: REAL*8 Units: [s]
T_SPEC_MAX	Description: Upper limit (longest period) for the definition of the response spectrum. Type: REAL*8 Units: [s]
T_CONT_MIN	Description: Shortest period for generating the frequency content of the motion. If 0.0, it adopts the value of T_SPEC_MIN. Type: REAL*8 Units: [s]
T_CONT_MAX	Description: Longest period for the frequency content. If 0.0, it adopts the value of T_SPEC_MAX. Type: REAL*8 Units: [s]
RFreqInc	Description: Relative logarithmic increment for frequency discretization. A smaller value generates more sinusoids. If 0.0, the program uses a default value. Type: REAL*8 Units: [-]

A.1.3 Block 3: Envelope Function and Duration

Parameter	Description and Details
ENV_TYPE	Description: Code that selects the type of intensity envelope function. Type: INTEGER*4 Values: 1 (None), 2 (Trapezoidal), 3 (Exponential), 4 (Compound).
T_RISE_ENV	Description: Rise time for the Trapezoidal and Compound envelopes. Type: REAL*8 Units: [s]

Parameter	Description and Details
T_PLATEAU_ENV	Description: Duration of the plateau for the Trapezoidal and Compound envelopes. Type: REAL*8 Units: [s]
DURATION_SIGNAL	Description: Total duration of the accelerogram to be generated. Type: REAL*8 Units: [s]

A.1.4 Block 4: Specific Envelope Parameters

Parameter	Description and Details
PARAM_POW_CMPD	Description: Exponent (n) for the rise phase of the Compound envelope (ENV_TYPE=4). Type: REAL*8 Units: [-]
PARAM_A0_EXP	Description: Amplitude factor (a_0) for the Exponential envelope (ENV_TYPE=3). Type: REAL*8 Units: [-]
PARAM_ALPHA_EXP	Description: Exponent α (rise in Exp. envelope, decay in Compound envelope). Type: REAL*8 Units: [-]
PARAM_BETA_EXP	Description: Exponent β (decay in Exp. envelope). Must satisfy $\alpha > \beta > 0$. Type: REAL*8 Units: [-]

A.1.5 Block 5: Accelerogram and Gravity Parameters

Parameter	Description and Details
DELTA_T_SIGNAL	Description: Time step of the generated accelerogram. If 0.0, a default value is used. Type: REAL*8 Units: [s]

Parameter	Description and Details
PGA_TARGET	Description: Target PGA value. If > 0 , the program adjusts the accelerogram to meet it. Type: REAL*8 Units: [DIST_UNIT/s ²]
G_ACCEL	Description: Value of the acceleration of gravity for calculating Arias Intensity. Type: REAL*8 Units: [DIST_UNIT/s ²]

A.1.6 Block 6: Simulation Control (Phase 1)

Parameter	Description and Details
SEED_RANDOM	Description: Initial seed for the random number generator. Type: INTEGER*4
N_DAMP_VALS	Description: Number of damping ratios to be analyzed. Type: INTEGER*4
N_ITERATIONS	Description: Maximum number of iterations for the spectral adjustment in Phase 1. Type: INTEGER*4
N_MOTIONS_GEN	Description: Total number of different accelerograms to be generated in Phase 1. Type: INTEGER*4
N_PERIODS_SPECT	Description: Number of period points for the spectrum calculation. Must be > 10 . Type: INTEGER*4
N_PTS_TARGET_SV	Description: Number of points defining the target spectrum. Must be ≥ 3 . Type: INTEGER*4

A.1.7 Block 7: Damping

Parameter	Description and Details
DAMP_RATIOS	Description: List of damping ratios (e.g., 0.05). The first one is the primary for the adjustment. Type: REAL*8 (Array)

A.1.8 Block 8: Optimization Control (Phase 2)

Parameter	Description and Details
MAX_STATE_TO_RUN	Description: Controls the level of refinement in Phase 2. A value of 0 skips this phase, while 5 (maximum) applies all optimization strategies. Type: INTEGER*4 Values: 0–5.
IMPROVE_THRS	Description: Stagnation threshold (in %) for Phase 2. The algorithm transitions to the next optimization state if the improvement in one sweep is less than this value. Type: REAL*8 Units: [%] Default value: 0.05%.

A.1.9 Block 9: Target Spectrum Definition

Parameter	Description and Details
ICODE_TARGET_TYPE	Description: Code specifying the type of target spectrum. Type: INTEGER*4 Values: 0 (PSA), 1 (PSV).
T_SV_TARGET, SV_TARGET_VALS	Description: Pairs (Period, Value) that define the target spectrum. Periods must be strictly increasing. Type: REAL*8 (two per line).

A.2 Output File Parameters

The main numerical values presented in the output file are described below.

A.2.1 Convergence Metrics

Throughout the execution, the program reports a set of four metrics to quantify the quality of the spectral fit. For a set of N spectral points, the normalized difference at each point i is first defined as:

$$d_i = \frac{|S_{target,i} - S_{calculated,i}|}{\max(|S_{target}|)} \times 100 \quad [\%]$$

From this set of differences $\{d_1, d_2, \dots, d_N\}$, the following metrics are calculated:

- **NORMALIZED MEAN DIFFERENCE [%] ($\bar{d}$):** This is the arithmetic mean of the normalized differences. It indicates the average deviation of the fit.

$$\bar{d} = \frac{1}{N} \sum_{i=1}^N d_i$$

- **NORMALIZED STANDARD DEVIATION [%] (σ_d):** This is the standard deviation of the normalized differences. It measures the variability or consistency of the difference across the periods.

$$\sigma_d = \sqrt{\frac{1}{N} \sum_{i=1}^N (d_i - \bar{d})^2}$$

- **NORMALIZED MEAN DIFFERENCE - TOP 5 [%] (d_{T5}):** This is the average of the top 5% of the normalized differences. It replaces a single “maximum difference” as a robust indicator of the largest discrepancies.
- **EQUIVALENT / SELECTED DIFFERENCE [%] (d_{eq}):** This is a combined and weighted metric derived from the previous three, used internally to identify the best result. It is calculated as:

$$d_{eq} = 0.35 \cdot \bar{d} + 0.40 \cdot \sigma_d + 0.25 \cdot d_{T5}$$

These metrics are presented at different stages of the process:

PHASE 1 - Summary per Motion. At the end of the frequency-domain optimization for each generated motion, the four metrics corresponding to the best iteration cycle found are presented.

PHASE 2 - Summary per Spectral Sweep. During the time-domain optimization, the four metrics are again presented to report the progress at the end of each complete spectral sweep.

A.2.2 Final Motion Parameters

Parameter	Description
PEAK GROUND ACCELERATION (FINAL)	PGA value of the final earthquake, after all optimization and scaling phases.
PEAK GROUND ACCELERATION (BEFORE SCALE)	PGA of the accelerogram before the scaling that is performed in Phase 1 (prior to the iterative loop). Allows evaluation of the initial scaling factor applied.
MAXIMUM GROUND VELOCITY	PGV value of the final earthquake.
MAXIMUM GROUND DISPLACEMENT	PGD value of the final earthquake.
ARIAS INTENSITY (IA)	Arias Intensity calculated for the final earthquake.
SIGNIFICANT DURATION (D5-95)	Significant duration of the final motion (5%-95% interval of the AI).

A.2.3 Columns of the Final Spectra Table

Column	Description
#	Sequence number of the spectral point.
PERIOD [s]	Vibration period of the SDOF oscillator.
FREQ. [Hz]	Vibration frequency of the SDOF oscillator, inverse of the period.
TARGET SV	Target pseudo-velocity (PSV) spectrum value interpolated at the calculation point.
CALC SV	Calculated PSV value of the final accelerogram.
DIFFER [%]	Percentage difference between the calculated and target PSV: $((S_{calculated}/S_{target}) - 1) \times 100$.
DISPL.SPECT	Displacement spectrum (SD) of the final accelerogram.
ACCEL.SPECT	Pseudo-acceleration spectrum (PSA) of the final accelerogram.
TIME MAX [s]	Time instant at which the maximum relative displacement of the oscillator occurs.

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Appendix B

Practical Guide for the Construction of Code-Based Design Spectra

B.1 Introduction: The Philosophy of the Design Spectrum

The design spectrum provided by a seismic code does not represent a real earthquake, but rather an **envelope of the seismic demand** with a uniform probability of exceedance for a given hazard level. Its purpose is to provide a standardized, simplified, and conservative tool for structural design. Most modern codes base their spectra on the concept of the **Uniform Hazard Spectrum (UHS)**, which is obtained from probabilistic seismic hazard assessment (PSHA) studies that consider all seismic sources that can affect a site. This appendix describes the procedure for constructing the **elastic design spectrum** for 5% damping using different codes, which is the one used as the target in the **earthquake** program.

Important Note on the Purpose of this Appendix

The objective of this appendix is to guide the user in constructing the **elastic response spectrum**. This spectrum represents the total seismic demand, without applying any reduction for ductility or nonlinear behavior. As discussed in detail in Subsection 4.1.3 of Chapter 4, it is this elastic spectrum that must be used as the target in the **earthquake_2025** program. The purpose is to generate a time history that contains the complete energy demand, which will subsequently be applied to a structural model (potentially nonlinear) in a time-history analysis. Therefore, in the following sections, when calculating the ordinates of the elastic spectrum, all response modification factors (such as “ q ” in Eurocode 8 or “ R ” in ASCE 7) must be taken with a value of unity or not be applied, as appropriate for each code.

B.2 General Methodology and Common Parameters

Despite differences in their formulation, almost all codes construct the design spectrum from a series of common parameters:

1. **Seismic Zonation:** The national territory is divided into zones of different hazard levels, each of which is assigned a base acceleration parameter.
2. **Site Classification:** The geotechnical profile is classified to account for local amplification effects.
3. **Importance Factor:** Magnifies the seismic action based on the occupancy and criticality of the structure.
4. **Spectral Shape:** The shape is defined by characteristic periods that mark the transitions between the acceleration-controlled, velocity-controlled, and displacement-controlled zones.

B.3 Eurocode 8 (EN 1998) [23]

B.3.1 Required Input Data

- **Reference peak ground acceleration on rock (a_{gR}):** From the zonation map of the corresponding National Annex.
- **Ground type:** A, B, C, D, or E, according to the geotechnical profile.
- **Spectrum type:** Choose between Type 1 (generally for earthquakes of magnitude $M_s > 5.5$) or Type 2.
- **Importance factor (I_E):** According to the building's importance class (I to IV).

B.3.2 Key Parameters and Their Meaning

- $a_g = I_E \cdot a_{gR}$: Design ground acceleration on rock, adjusted for importance.
- S : Soil amplification factor.
- T_B, T_C, T_D : Characteristic periods defining the limits of the spectrum regions.

B.3.3 Step-by-Step Calculation Procedure

Note on the elastic spectrum: To obtain the elastic spectrum, the behavior factor q must be taken as $q = 1.0$.

1. **Step 1:** Obtain a_{gR} from the zonation map.
2. **Step 2:** Determine the Ground Type and Spectrum Type for the location.

3. **Step 3:** With this data, obtain the values of S , T_B , T_C , and T_D from the code's tables.
4. **Step 4:** Select the importance factor I_E and calculate $a_g = I_E \cdot a_{gR}$.
5. **Step 5:** Construct the spectral ordinate $S_e(T)$ for the elastic spectrum ($q = 1.0$) according to the following expressions:

$$S_e(T) = \begin{cases} a_g \cdot S \cdot \left[1 + \frac{T}{T_B}(2.5 - 1)\right] & \text{if } 0 \leq T < T_B \\ a_g \cdot S \cdot 2.5 & \text{if } T_B \leq T \leq T_C \\ a_g \cdot S \cdot 2.5 \cdot \left(\frac{T_C}{T}\right) & \text{if } T_C \leq T \leq T_D \\ a_g \cdot S \cdot 2.5 \cdot \left(\frac{T_C T_D}{T^2}\right) & \text{if } T > T_D \end{cases} \quad (\text{B.1})$$

Figure B.1 shows a typical example. For an in-depth treatment of seismic design of concrete buildings according to this code, the reader is referred to [24].

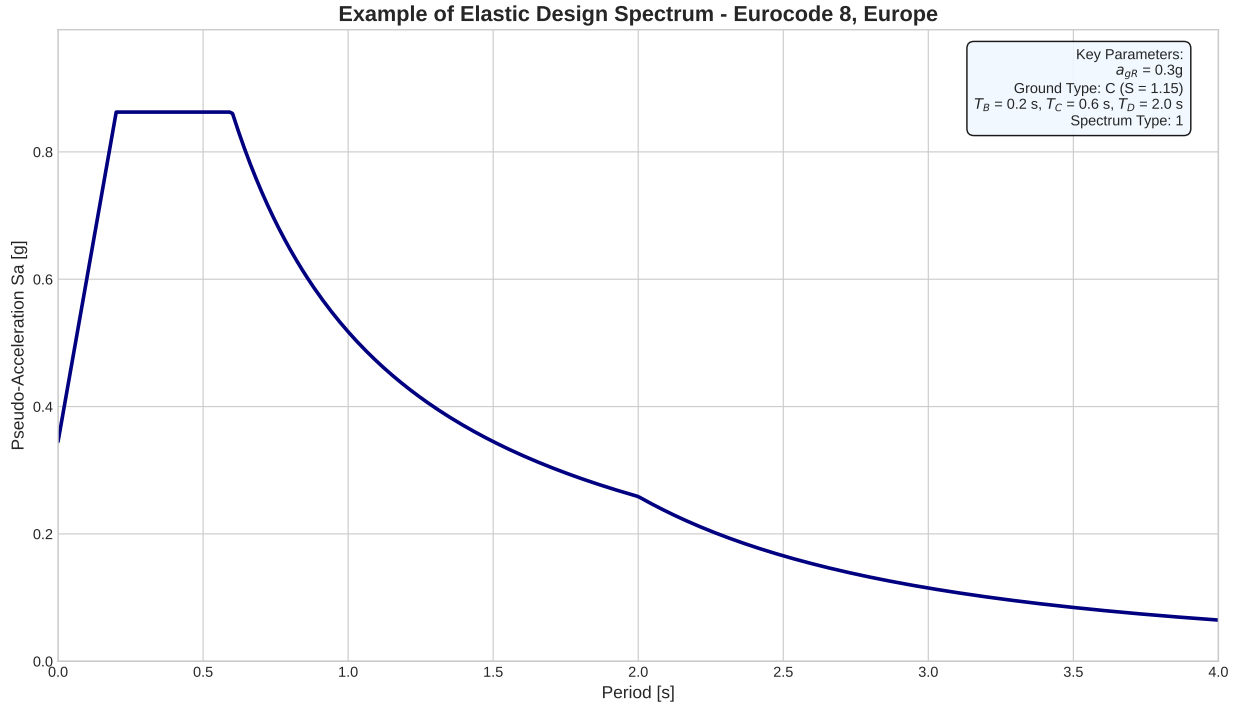


Figure B.1: Example of an Elastic Design Spectrum according to Eurocode 8.

B.4 United States (ASCE 7) [4]

B.4.1 Required Input Data

- **Spectral parameters on rock:** S_S (acceleration for short period) and S_1 (acceleration for a period of 1.0s), obtained from seismic hazard maps.
- **Site Class:** A, B, C, D, E, or F, determined from the geotechnical profile.

B.4.2 Key Parameters and Their Meaning

- F_a, F_v : Site factors for short and long periods, respectively.
- S_{DS}, S_{D1} : Design spectral parameters, adjusted for site effects.
- T_0, T_S : Periods defining the start and end of the spectrum's plateau.

B.4.3 Step-by-Step Calculation Procedure

Note on the elastic spectrum: The following formulas construct the elastic design spectrum. The response modification factor, R , should not be applied in the definition of the seismic action.

1. **Step 1:** Determine the site factors F_a and F_v from tables 11.4-1 and 11.4-2 of ASCE 7, based on the Site Class and the values of S_S and S_1 .
2. **Step 2:** Calculate the spectral accelerations of the Maximum Considered Earthquake (MCE_R), adjusted for site effects:

$$S_{MS} = F_a S_S \quad (B.2)$$

$$S_{M1} = F_v S_1 \quad (B.3)$$

3. **Step 3:** Calculate the design spectral accelerations, which correspond to 2/3 of the MCE_R :

$$S_{DS} = \frac{2}{3} S_{MS} \quad (B.4)$$

$$S_{D1} = \frac{2}{3} S_{M1} \quad (B.5)$$

4. **Step 4:** Calculate the periods that define the shape of the spectrum:

$$T_0 = 0.2 \frac{S_{D1}}{S_{DS}} \quad (B.6)$$

$$T_S = \frac{S_{D1}}{S_{DS}} \quad (B.7)$$

5. **Step 5:** Construct the ordinate of the elastic design spectrum $S_a(T)$ for each period T :

$$S_a(T) = \begin{cases} S_{DS} \left(0.4 + 0.6 \frac{T}{T_0} \right) & \text{if } 0 \leq T < T_0 \\ S_{DS} & \text{if } T_0 \leq T \leq T_S \\ S_{D1}/T & \text{if } T_S < T \leq T_L \\ S_{D1}T_L/T^2 & \text{if } T > T_L \end{cases} \quad (\text{B.8})$$

where T_L is the long-period transition period, obtained from maps.

Figure B.2 shows a typical spectrum.

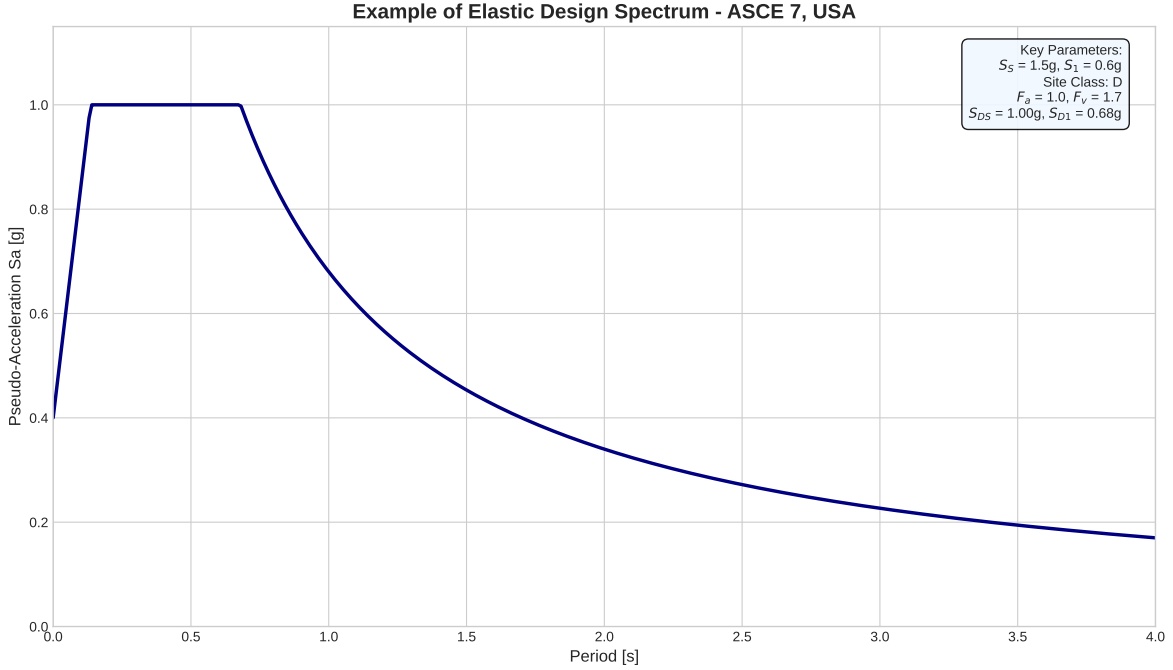


Figure B.2: Example of an Elastic Design Spectrum according to ASCE 7 of the U.S.A..

B.5 Canada (NBCC) [39]

B.5.1 Required Input Data

- **UHS ordinates for rock (Class C):** $S_a(0.2)$, $S_a(0.5)$, $S_a(1.0)$, $S_a(2.0)$, obtained for the specific location.
- **Site Class:** A-F. **Importance factor** (I_E).

B.5.2 Key Parameters and Their Meaning

- $S_a(T)$: Spectral ordinate for period T , obtained from seismic hazard maps.
- F_a, F_v : Site amplification factors for short and long periods.

B.5.3 Step-by-Step Calculation Procedure

Note on the elastic spectrum: This procedure defines the elastic Design Spectral Acceleration. The reduction factors for ductility and overstrength (R_d, R_o) should not be applied in the definition of the seismic action.

1. **Step 1:** Obtain the values of $S_a(0.2)$, $S_a(0.5)$, $S_a(1.0)$, and $S_a(2.0)$ for the location.
2. **Step 2:** Determine the site factors F_a (a function of $S_a(0.2)$) and F_v (a function of $S_a(1.0)$) from the code's tables.
3. **Step 3:** Calculate the five key points of the design spectrum $S(T) = I_E \cdot S_{mod}(T)$, where $S_{mod}(T)$ is obtained as follows:
 - $S(0.2) = F_a S_a(0.2)$
 - $S(0.5) = F_v S_a(0.5)$
 - $S(1.0) = F_v S_a(1.0)$
 - $S(2.0) = F_v S_a(2.0)$
4. **Step 4:** Construct the complete spectrum by linearly interpolating between the calculated key points. For undefined intermediate periods, linear interpolation is done on a logarithmic scale. For periods $T > 2.0$ s, interpolation continues with the values of $S_a(5.0)$ and $S_a(10.0)$ if available. The constant acceleration plateau extends from $T = 0$ to $T = 0.2$ s with the value of $S(0.2)$.

Figure B.3 shows a typical example.

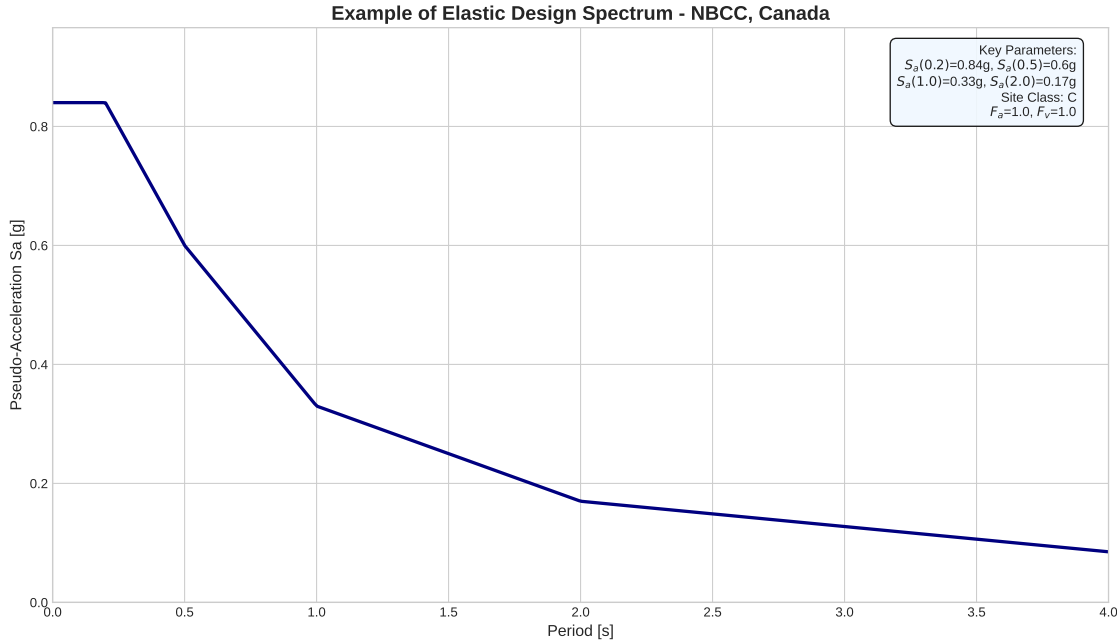


Figure B.3: Example of an Elastic Design Spectrum according to NBCC of Canada.

B.6 Japan (BSL) [11]

B.6.1 Required Input Data

- **Seismic zone factor (Z):** Varies from 0.7 to 1.0.
- **Soil type (I, II, or III):** Defines the characteristic period T_C .
- **Earthquake level:** Base shear coefficient, C_0 , of 1.0 for severe earthquakes.

B.6.2 Key Parameters and Their Meaning

- C_0 : Standard base shear coefficient (represents the earthquake level).
- Z : Zonal hazard adjustment factor.
- R_t : Dynamic spectral amplification factor, dependent on period.
- A_i : Vertical distribution factor of the seismic force.

B.6.3 Step-by-Step Calculation Procedure

Note on the elastic spectrum: The use of a base shear coefficient $C_0 = 1.0$ corresponds to the definition of the demand for a severe earthquake considering an elastic response. This value of $C_0 = 1.0$ must always be used at this stage.

1. **Step 1:** Determine Z and the Soil Class for the location. Obtain the period T_C associated with the soil class (e.g., $T_C = 0.64\text{s}$ for Soil Type II).
2. **Step 2:** For the elastic spectrum of a severe earthquake, use $C_0 = 1.0$.
3. **Step 3:** Construct the spectral ordinate $S_a(T)$ (normalized by g , assuming $A_i = 1.0$ and $G_s = 1.0$):

$$S_a(T) = \begin{cases} Z \cdot C_0 \cdot 2.5 & \text{if } 0 \leq T \leq T_C \\ Z \cdot C_0 \cdot 2.5 \cdot \left(\frac{T_C}{T}\right) & \text{if } T > T_C \end{cases} \quad (\text{B.9})$$

Note: Some implementations define a transition ramp for very short periods, but the most simplified and commonly referenced form of the standard extends the constant acceleration plateau to $T=0$.

Figure B.4 shows a typical example.

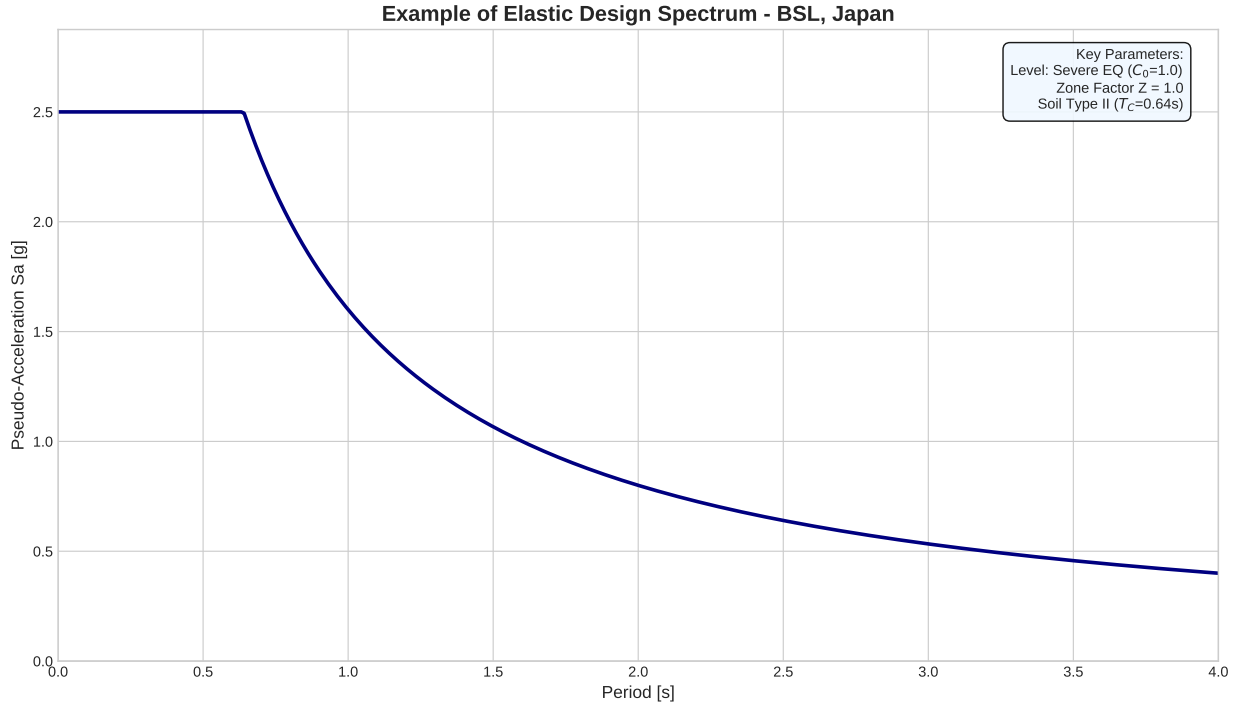


Figure B.4: Example of an Elastic Design Spectrum according to BSL of Japan.

B.7 Chile (NCh433 / DS61) [41]

B.7.1 Required Input Data

- **Seismic zone (1, 2, or 3):** Defines the maximum effective acceleration A_0 ($0.20g$, $0.30g$, or $0.40g$, respectively).
- **Soil type (A, B, C, D, or E):** Classification according to D.S. N°61 (based on V_{s30} , RQD, etc.), which determines parameters S , T_0 , and exponent p .
- **Building category (I to IV):** Defines the importance factor I .

B.7.2 Key Parameters and Their Meaning

- A_0 : Maximum effective ground acceleration on rock.
- I : Building importance factor.
- S : Soil type multiplier factor.
- T_0, p : Soil deposit parameters governing the spectral shape and decay rate.
- $\alpha(T)$: Dynamic spectral amplification factor dependent on period T .

B.7.3 Step-by-Step Calculation Procedure

Note on the elastic spectrum: The elastic spectral ordinate is obtained directly from these equations. The response reduction factor R^* should not be applied in this step ($R^* = 1.0$). Unlike other codes that define a constant horizontal plateau for intermediate periods, the Chilean standard features a smooth, continuous amplification curve.

1. **Step 1:** Obtain acceleration A_0 from the seismic zone and the importance factor I according to the building occupancy category.
2. **Step 2:** Determine the Soil Type and extract parameters S , T_0 , and exponent p from the code tables.
3. **Step 3:** Calculate the elastic design spectral ordinate $S_a(T) = I \cdot S \cdot A_0 \cdot \alpha(T)$, where the continuous dynamic amplification factor $\alpha(T)$ is evaluated as:

$$\alpha(T) = \frac{1 + 4.5 (T/T_0)^p}{1 + (T/T_0)^3} \quad (\text{B.10})$$

This function reaches a maximum amplification close to $\alpha_{\max} \approx 2.75$ around T_0 and decays smoothly toward longer periods, without presenting a strict horizontal plateau.

Figure B.5 shows a typical example.

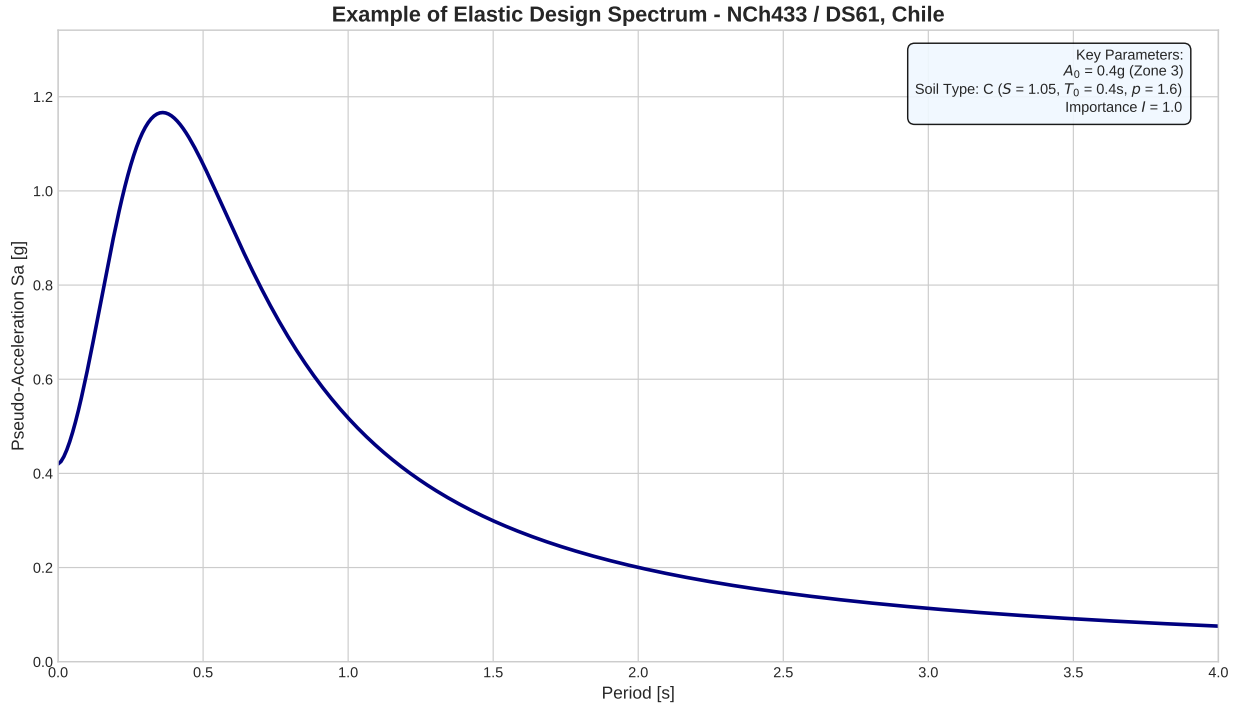


Figure B.5: Example of Elastic Design Spectrum according to NCh433 / DS61 of Chile.

B.8 Mexico (CFE-MDOC) [14]

B.8.1 Required Input Data

- Regional parameters a_0, c, T_a, T_b, r obtained from the Design Seismic Actions System (SASID).

B.8.2 Key Parameters and Their Meaning

- a_0 : Ordinate at the origin.
- c : Plateau ordinate.
- T_a, T_b : Start and end periods of the plateau.
- r : Decay exponent.

B.8.3 Step-by-Step Calculation Procedure

Note on the elastic spectrum: The formulas construct the regional design spectrum, which represents the elastic demand. Factors for ductility (Q') or others should not be applied at this stage.

1. **Step 1:** Obtain the five parameters for the site of interest.
2. **Step 2:** Construct the design spectrum $S_a(T)$ (in m/s^2):

$$S_a(T) = \begin{cases} a_0 + (c - a_0)\frac{T}{T_a} & \text{if } 0 \leq T < T_a \\ c & \text{if } T_a \leq T \leq T_b \\ c\left(\frac{T_b}{T}\right)^r & \text{if } T > T_b \end{cases} \quad (\text{B.11})$$

3. **Step 3:** To express in units of g , divide by $g \approx 9.81 \text{ m/s}^2$.

Figure B.6 shows a typical example.

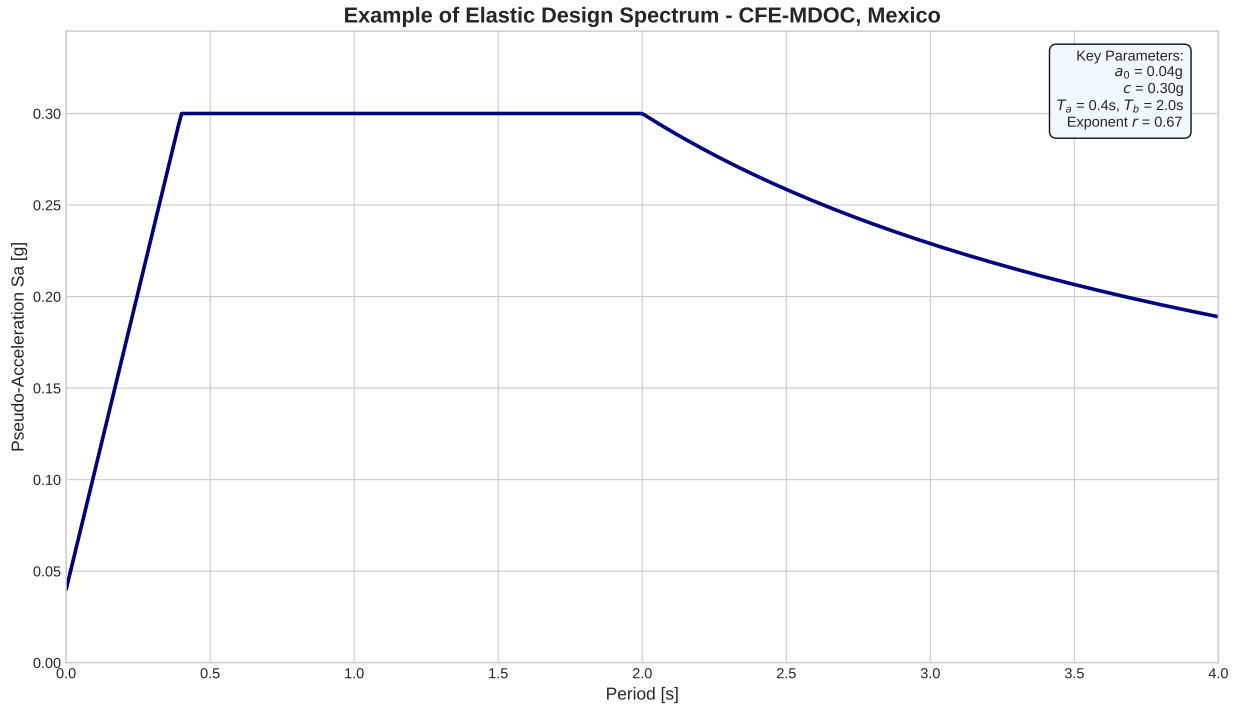


Figure B.6: Example of an Elastic Design Spectrum according to CFE-MDOC of Mexico.

B.9 Peru (NTE E.030) [45]

B.9.1 Required Input Data

- **Zone factor** (Z), **Soil profile** (S), **Use factor** (U), and **Periods** (T_P, T_L).

B.9.2 Key Parameters and Their Meaning

- Z, U, S : Zone, use, and soil factors that scale the acceleration.
- C : Dynamic amplification factor.
- T_P, T_L : Start and end periods of the amplification spectrum's plateau.

B.9.3 Step-by-Step Calculation Procedure

Note on the elastic spectrum: The elastic spectrum ordinate is calculated according to the following expression. The ductility reduction factor R should not be applied.

1. **Step 1:** Determine Z, S, U, T_P, T_L from the code's tables.
2. **Step 2:** Calculate the seismic amplification factor C for each period T :

$$C = \begin{cases} 2.5 & \text{if } T \leq T_P \\ 2.5 \left(\frac{T_P}{T} \right) & \text{if } T_P < T \leq T_L \\ 2.5 \left(\frac{T_P T_L}{T^2} \right) & \text{if } T > T_L \end{cases} \quad (\text{B.12})$$

3. **Step 3:** Calculate the spectral ordinate: $S_a(T) = Z \cdot U \cdot C \cdot S$.

Figure B.7 shows a typical example.

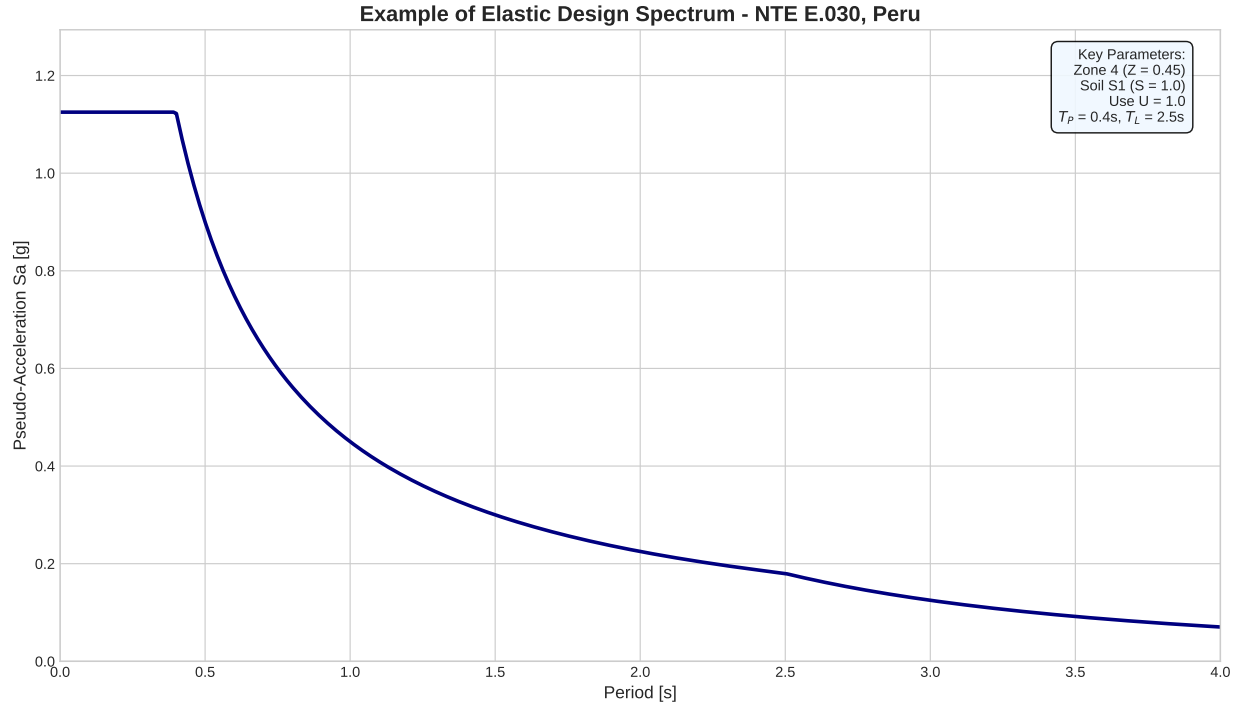


Figure B.7: Example of an Elastic Design Spectrum according to NTE E.030 of Peru.

B.10 Bolivia (NBDS-2023) [40]

B.10.1 Required Input Data

- **Acceleration on rock (S_0):** From the seismic zonation map.
- **Soil class:** S0 to S5.

B.10.2 Key Parameters and Their Meaning

- S_0 : Peak ground acceleration on rock.
- F_a, F_v : Site factors for short and long periods.
- T_0, T_s, T_L : Characteristic periods that define the spectral shape.

B.10.3 Step-by-Step Calculation Procedure

Note on the elastic spectrum: The notation $S_{ae}(T)$ explicitly refers to the elastic pseudo-acceleration spectrum. The response modification factor R should not be applied to define the action.

1. **Step 1:** Obtain S_0 from the map for the site of interest.
2. **Step 2:** Determine the Soil Class and obtain the site factors F_a and F_v from Tables 3 and 4 of the code.
3. **Step 3:** Calculate the characteristic periods of the spectrum:

$$T_0 = 0.15 \frac{F_v}{F_a} \quad (\text{B.13})$$

$$T_s = 0.5 \frac{F_v}{F_a} \quad (\text{B.14})$$

The long-period transition period, T_L , is obtained from the seismic hazard maps provided by the code.

4. **Step 4:** Construct the elastic pseudo-acceleration spectrum $S_{ae}(T)$:

$$S_{ae}(T) = \begin{cases} F_a S_0 \left(1 + 1.5 \frac{T}{T_0}\right) & \text{if } T < T_0 \\ 2.5 F_a S_0 & \text{if } T_0 \leq T \leq T_s \\ 1.25 F_v S_0 / T & \text{if } T_s < T \leq T_L \\ 1.25 F_v S_0 T_L / T^2 & \text{if } T > T_L \end{cases} \quad (\text{B.15})$$

Figure B.8 shows a typical example.

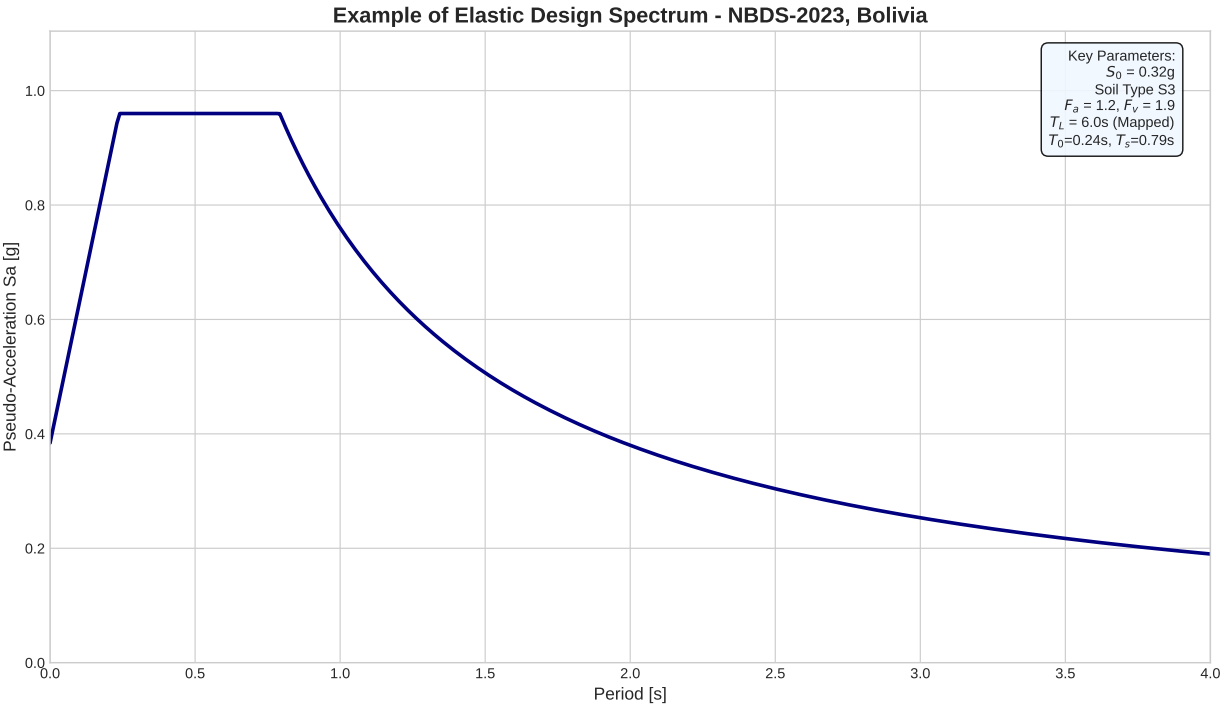


Figure B.8: Example of an Elastic Design Spectrum according to NBDS-2023 of Bolivia.

Appendix C

Case Study: From Code-Based Spectrum to Structural Analysis

C.1 Objectives and Scope of the Case Study

This appendix serves a dual purpose. The **general objective** is to illustrate the complete workflow that a structural engineer would undertake for the seismic evaluation of a structure, thus connecting the theory from the previous chapters with a practical application. As discussed in Chapter 7, multiple analysis methods exist, with none representing a universal solution.

The **specific objective** of this appendix is, through a specific case study, to demonstrate the coherence and consistency that exists among the different analysis methods when they are applied to a system with linear behavior.

The process is summarized in the following stages:

1. **Step 1: Definition of the Seismic Hazard:** An elastic design spectrum is constructed for a specific site, following the guidelines of the American standard ASCE 7. This spectrum will be the basis for the Modal Response Spectrum Analysis (MRSA) and for the generation of an artificial earthquake for the time-history analyses (MTHA, EDIA, and IDIA).
2. **Step 2: Generation of the Time-History Load:** The `earthquake_2025` program is used to generate a “super-compatible” artificial displacement history from the aforementioned spectrum (see the “recommendation for time-history analysis” given in paragraph 4.1.1 on page 39). The result of the two-phase methodology will be shown, highlighting the high level of fit achieved, to obtain the load signal to be applied in the time-domain analyses.
3. **Step 3: Structural Modeling:** A finite element model of a multi-story reinforced concrete building is created in the commercial advanced analysis software **Abaqus** [1, 20] (this example was sized so that the free learning version of **Abaqus** can be used).

4. **Step 4: Analysis and Cross-Validation:** Four types of linear analyses will be performed on the model:
 - Modal Response Spectrum Analysis (MRSA).
 - Modal Time-History Analysis (MTHA).
 - Implicit Direct-Integration Analysis (IDIA).
 - Explicit Direct-Integration Analysis (EDIA).
5. **Comparison and Conclusions:** Finally, the results of the four analyses are compared to verify that, in the elastic range, they all lead to very similar responses. This validates not only the consistency of the numerical methods but also the ability of the `earthquake_2025` program to generate a signal that contains the damage potential and energy demand implicit in the elastic design spectrum.

C.2 Step 1: Definition of the Design Spectrum (ASCE 7)

For this case study, a hypothetical site in San Diego, California, with a Site Class D (Stiff Soil) classification has been selected. Using the seismic hazard mapping tools provided by the U.S. Geological Survey (USGS) [59] and the procedures of the ASCE 7 standard [4], the spectral parameters defining the hazard are obtained. Figure C.1 shows the resulting elastic design acceleration spectrum, which will serve as the target spectrum for `earthquake_2025`. The spectrum's plateau is situated at approximately $0.6g = 5.89[m/s^2]$, representing a severe seismic demand.

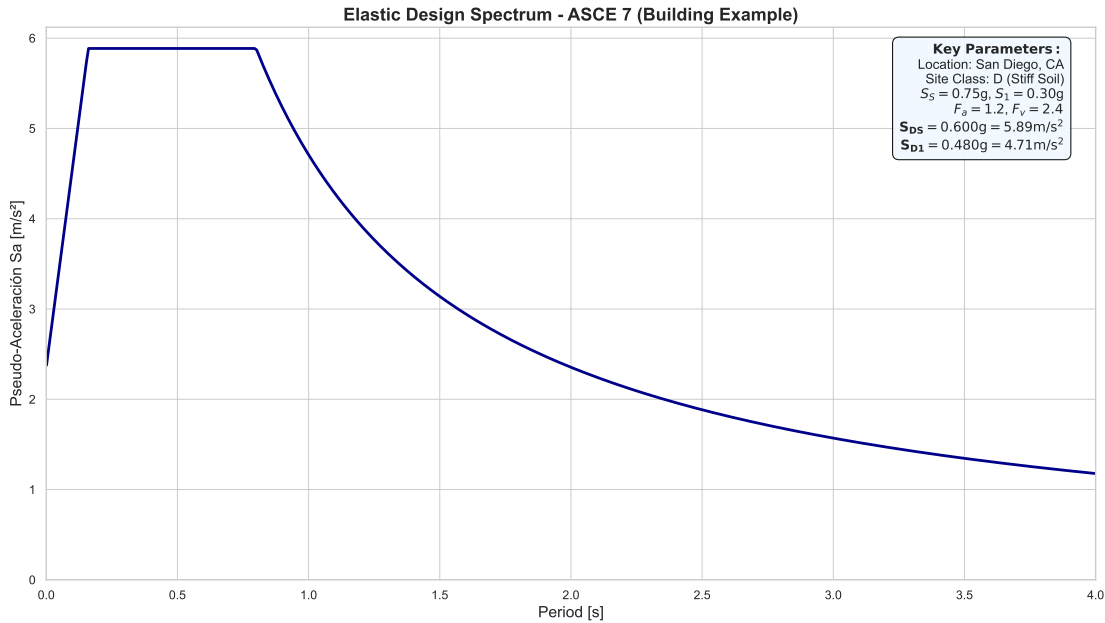


Figure C.1: Elastic Design Spectrum according to ASCE 7 for the case study site.

C.3 Step 2: Generation of the “Super-Compatible” Time History

With the design spectrum defined, the next step is to generate a high-quality time history using the two-phase process of `earthquake_2025`. To do this, the program is configured and run with the appropriate input parameters.

C.3.1 Input File for `earthquake_2025`

Figure C.2 shows the complete input file used for this simulation, which defines a 13-point spectrum, a duration of 20 seconds, and a trapezoidal envelope. To force convergence towards an optimal candidate in Phase 1, a high number of iterations (`N_ITERATIONS=160`) was set. Subsequently, for Phase 2, a rigorous improvement threshold was maintained (`IMPROVE_THRS=0.001%`) in order to force the algorithm to achieve a high level of compatibility; a value on the order of 0.05% is typically more than sufficient. Finally, the target PGA value was matched with the spectrum’s ordinate at $T = 0.0$ s, ensuring consistency in the short-period range.

```

Elastic Design Spectrum - ASCE 7 (Building Example)
| DISTANCE UNIT|
| [m]          |
| TSpecMin | TSpecMax | TContMin | TContMax | RFreqInc |
| 0.01      | 4.0      | 0.0      | 0.0      | 0.002    |
| ENV_TYPE | T_RISE  | T_PLATO  | DURATION |
| 2         | 1.0     | 18.0     | 20.0     |
| POW      | A0      | ALPHA    | BETA     |
| 0.0      | 0.0     | 0.0      | 0.0      |
| DELTA_T  | PGA     |
| 0.002    | 2.3544  |
| G_ACCEL  |
| 9.81     |
| ISEED    | NDAMP    | NITERA   | NMOTION  | NPERIOD  | NPTS-SPECT |
| 53536    | 1        | 160      | 10       | 300      | 13         |
| DAMP1    | DAMP2    | DAMP3    | DAMP4    | DAMP5    |
| 0.05     |          |          |          |          |
| MaxStates | IMPROVE_THRS |
| 5         | 0.001    |
| ICODE_SPECT_TYPE (0=SA_USER, 1=SV_USER)|
| 0         |
| T         | SA       |
| 0.0100    | 2.57512  |
| 0.1600    | 5.88600  |
| 0.8000    | 5.88600  |
| 1.1200    | 4.20429  |
| 1.4400    | 3.27000  |
| 1.7600    | 2.67545  |
| 2.0800    | 2.26385  |
| 2.4000    | 1.96200  |
| 2.7200    | 1.73118  |
| 3.0400    | 1.54895  |
| 3.3600    | 1.40143  |
| 3.6800    | 1.27957  |
| 4.0000    | 1.17720  |

```

Figure C.2: Input file used to generate the compatible artificial earthquake.

C.3.2 Results of the Artificial Earthquake Generation

The `earthquake_2025` program converges successfully, producing an artificial earthquake whose response spectrum fits the target spectrum excellently, as can be seen in Figure C.3. Figure C.4 shows the echo of the input parameters, and Figure C.5 shows an excerpt from the output file with the kinematic and intensity characteristics of the generated motion. The complete time histories of acceleration, velocity, and displacement are presented in Figures C.6, C.7, and C.8, respectively.

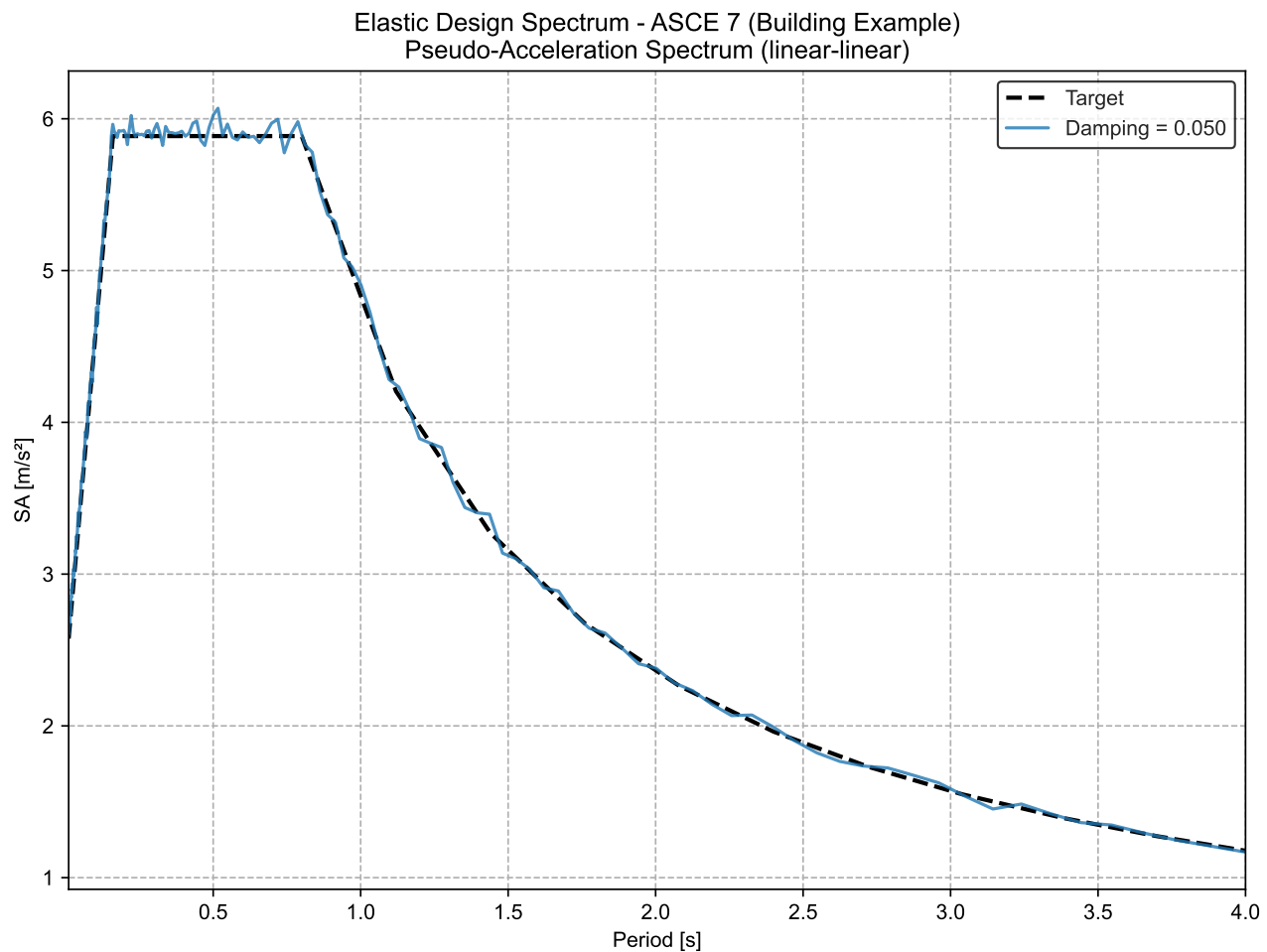


Figure C.3: Linear comparison between the target ASCE 7 acceleration spectrum (dashed black line) and the calculated spectrum of the generated accelerogram (blue line).

Note in the figure the slight *ripple* that the calculated spectrum (blue line) exhibits along the horizontal plateau of the target spectrum. This behavior is not a flaw in the algorithm, but rather the direct consequence of the physical-mathematical limitation described in Section 3.7. As demonstrated there, a perfectly flat and horizontal spectrum is physically unrealizable for a single time history. The observed undulations are, therefore, the signature of a physically plausible solution. Consequently, in this plateau region, the objective of “super-compatibility” is not to eliminate these oscillations—which is impossible—but to minimize them.

```

TARGET SPECTRUM AND FREQUENCY CONTENT DEFINITION:
  TARGET SPECTRUM - SMALLEST PERIOD ..... 0.0100 [s]
  TARGET SPECTRUM - LARGEST PERIOD ..... 4.0000 [s]
  MOTION CONTENT - SMALLEST PERIOD ..... 0.0100 [s]
  MOTION CONTENT - LARGEST PERIOD ..... 4.0000 [s]
  FREQUENCY GENERATION LN INCREMENT ..... 0.0020 [-]

TIME HISTORY PARAMETERS:
  TIME INCREMENT FOR SIGNAL ..... 0.0020 [s]
  TOTAL DURATION OF SIGNAL ..... 20.0000 [s]
  NUMBER OF POINTS IN SIGNAL ..... 10001 [#]

ANALYSIS TARGETS AND RESOLUTION:
  TARGET PEAK GROUND ACCELERATION (PGA) ..... 2.3544 [m/s2]
  GRAVITATIONAL ACCELERATION - FOR ARIAS_I ... 9.8100 [m/s2]
  NO. OF PERIODS FOR RESPONSE SPECTRUM ..... 300 [#]

SIMULATION LOOP PARAMETERS:
  RANDOM NUMBER SEED (INITIAL) ..... 53536 [#]
  NO. OF DIFFERENT MOTIONS TO GENERATE ..... 10 [#]
  NO. OF ITERATIONS FOR SPECTRUM MATCHING .... 160 [#]

TIME-DOMAIN ADJUSTMENT PARAMETERS (PHASE 2):
  MAXIMUM OPTIMIZATION STATES TO RUN ..... 5 [0-5]
  TIME-DOMAIN IMPROVEMENT THRESHOLD ..... 0.0010 [%]

DAMPING RATIOS FOR SPECTRUM CALCULATION:
  NO. OF DIFFERENT DAMPING RATIOS ..... 1 [#]
  DAMP1=0.05

ENVELOPE FUNCTION DEFINITION:
  ENVELOPE FUNCTION TYPE (TRAPEZOIDAL) ..... 2 [#]
  INTENSITY ENVELOPE RISE TIME ..... 1.0000 [s]
  INTENSITY ENVELOPE PLATEAU DURATION ..... 18.0000 [s]

TARGET SPECTRUM:
  TARGET SPECTRUM TYPE (ICODE) ..... 0 (ACCELERATION)

NOTE: THE USER-SUPPLIED SPECTRUM HAS BEEN DENSIFIED
      FOR A PRECISE FIT TO ITS PIECEWISE-LINEAR SEGMENTS.

      NO. OF POINTS DEFINING TARGET SPECTRUM ..... 201 [#]

#   PERIOD      FREQUENCY    DISPL.SPECT.    VEL.SPECT.    ACCEL.SPECT.
   [s]         [Hz]         [m]           [m/s]        [m/s2]
1   0.01000     100.000        0.652286E-05   0.409843E-02  2.57512
2   0.03005     33.278         0.690255E-04   0.144325E-01  3.01768
.   .           .           .             .             .
.   .           .           .             .             .
.   .           .           .             .             .
200 3.98075     0.251          0.474992       0.749724      1.18336
201 4.00000     0.250          0.477101       0.749429      1.17720

PROPERTIES OF THE INITIAL THEORETICAL PSDF:
  PSDF_INITIAL CENTRAL ANGULAR FREQUENCY (f) .. 11.3977 [Hz]
  PSDF_INITIAL CENTRAL ANGULAR FREQUENCY (wc) . 71.6139 [rad/s]
  PSDF_INITIAL DISPERSION PARAMETER (Q) ..... 0.8509 [-]
  PSDF_INITIAL STD. DEV. OF ACCELERATIONS ..... 0.9603 [m/s2]

PROPERTIES OF THE DISCRETIZED CONTENT FOR SYNTHESIS:
  TOTAL POWER (INTEGRAL OF PSDF_CONTENT) ..... 0.9212 [m2/s3]
  ROOT-MEAN-SQUARE FREQUENCY ..... 11.4045 [Hz]
  ROOT-MEAN-SQUARE CIRCULAR FREQUENCY ..... 71.6565 [rad/s]
  PERIOD CORRESPONDING TO RMS FREQUENCY ..... 0.0877 [s]
  NUMBER OF FREQUENCIES FOR SYNTHESIS ..... 2999 [#]

```

Figure C.4: Fragment of the output file showing the echo of input parameters and the calculated properties of the initial PSDF.

```
#####
OVERALL BEST GROUND MOTION
#####

RESULTS FOR GENERATED GROUND MOTION NUMBER ..        6 [#]
SELECTED FROM FREQUENCY-DOMAIN ITERATION ....        152 [#]
USING INITIAL RANDOM SEED .....                     26860 [#]

METRICS AFTER FREQUENCY-DOMAIN OPTIMIZATION WITH SMOOTH PSDF:
  NORMALIZED MEAN DIFFERENCE - OVERALL .....        0.7444 [%]
  NORMALIZED MEAN DIFFERENCE - TOP 5% .....        3.9221 [%]
  NORMALIZED STANDARD DEVIATION .....              1.0838 [%]
  EQUIVALENT DIFFERENCE (BEST OF PHASE 1) ...        1.6746 [%]

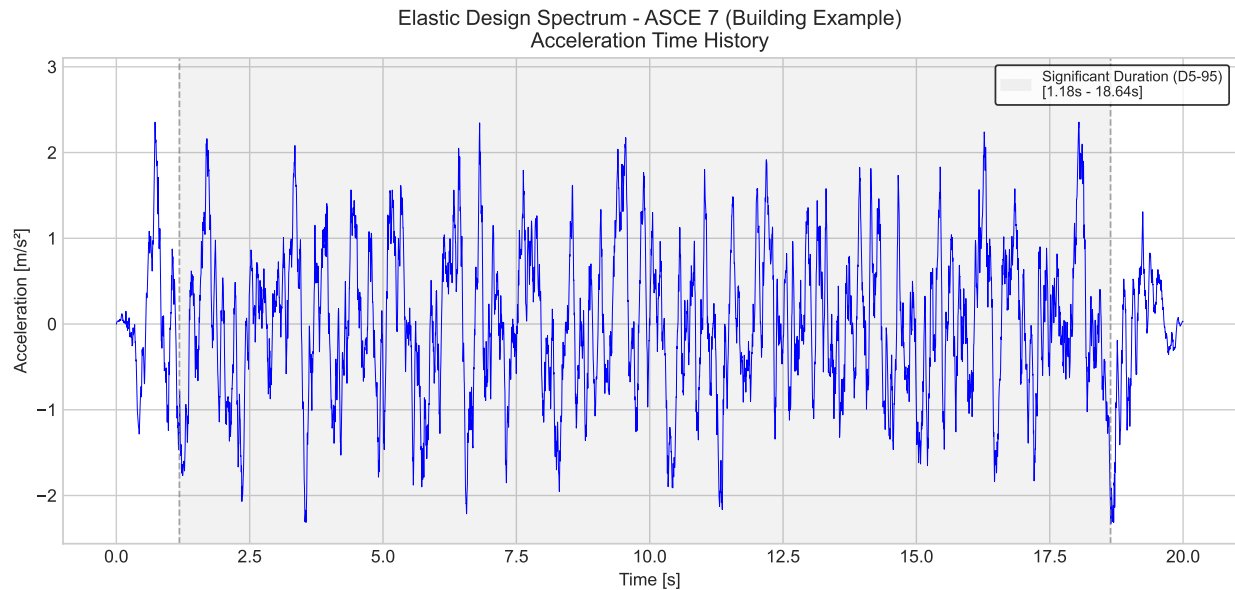
METRICS AFTER TIME-DOMAIN ADJUSTMENT WITH CORRECTIVE WAVELETS:
  NORMALIZED MEAN DIFFERENCE - OVERALL .....        0.4504 [%]
  NORMALIZED MEAN DIFFERENCE - TOP 5% .....        2.2645 [%]
  NORMALIZED STANDARD DEVIATION .....              0.6766 [%]
  EQUIVALENT DIFFERENCE (BEST OF PHASE 2) ...        0.9944 [%]

PEAK GROUND ACCELERATION (FINAL) .....             2.3544 [m/s2]
AT TIME .....                                       0.7260 [s]
PEAK GROUND ACCELERATION (BEFORE SCALE) .....       3.3736 [m/s2]
AT TIME .....                                       3.5360 [s]
MAXIMUM GROUND VELOCITY .....                     0.4017 [m/s]
MAXIMUM GROUND DISPLACEMENT .....                 0.3520 [m]

MAXIMUM PEAK FLOOR ACCELERATION (PFA) .....        6.0864 [m/s2]
OCCURRING AT PERIOD .....                         0.5078 [s]

ARIAS INTENSITY (IA) .....                        2.3280 [m/s]
  DESTRUCTIVE POTENTIAL (ID+) .....                1.2692 [m/s]
  DESTRUCTIVE POTENTIAL (ID-) .....                1.0588 [m/s]
  VERIFICATION (ID+ + ID-) .....                  2.3280 [m/s]
  NET ENERGY IMPULSE (ID+ - ID-) .....            0.2104 [m/s]
CUMULATIVE ABSOLUTE VELOCITY (CAV) .....           13.5578 [m/s]
HOUSNER INTENSITY (HI) .....                      1.5807 [m]
SIGNIFICANT DURATION (D5-95) .....                17.4560 [s]
```

Figure C.5: Summary of the final generated motion's characteristics.

Figure C.6: Acceleration time history of the generated motion ($a_g(t)$).

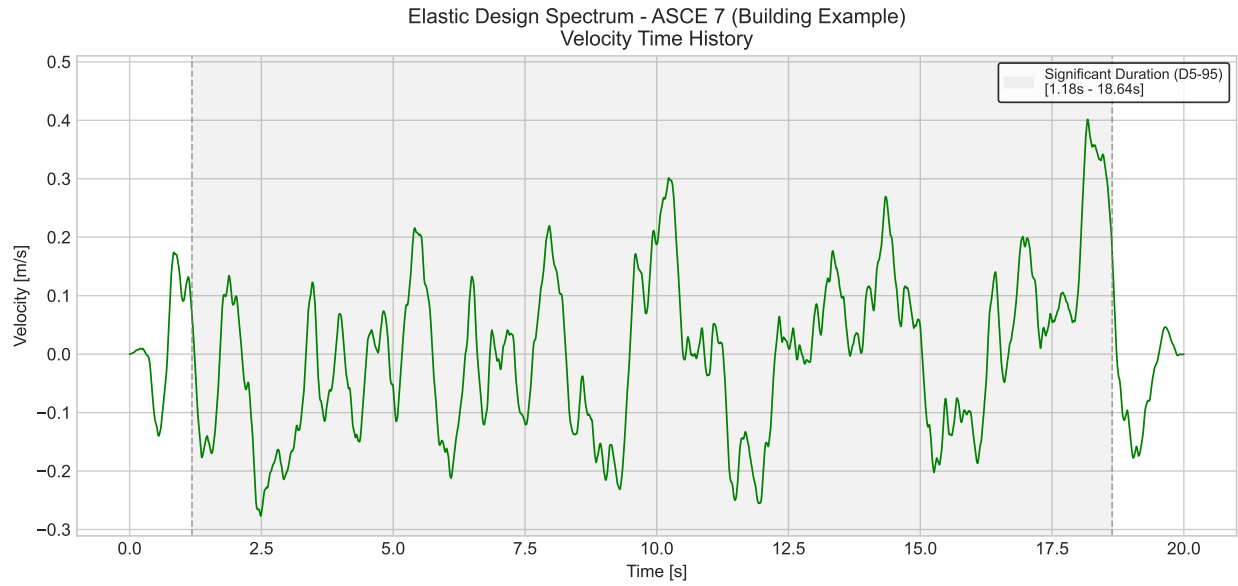


Figure C.7: Velocity time history of the generated motion ($v_g(t)$).

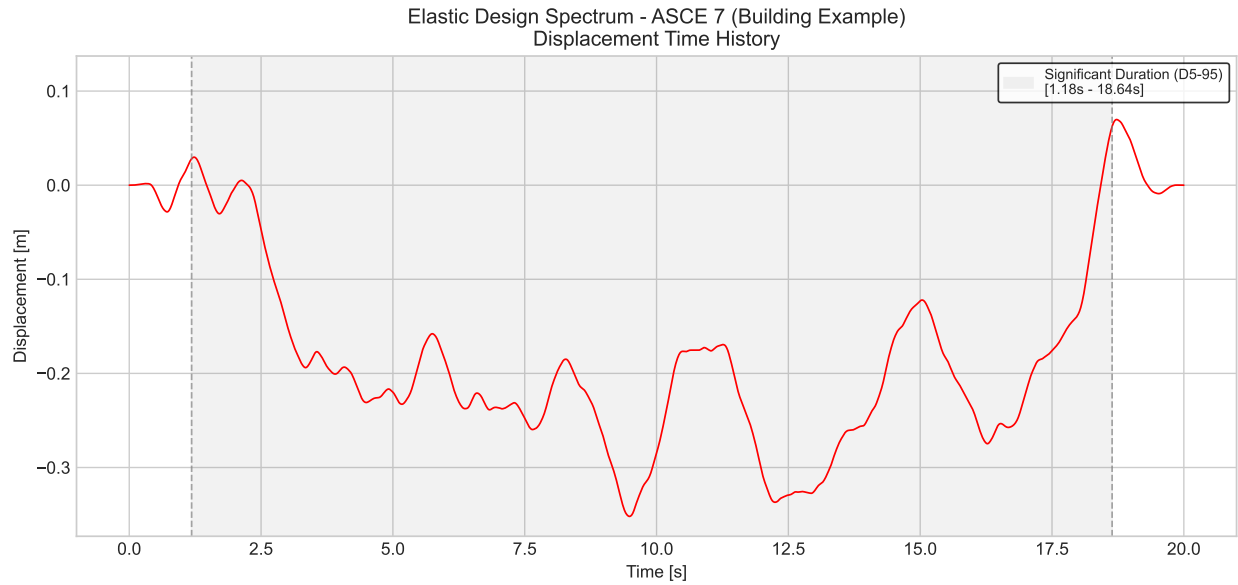


Figure C.8: Displacement time history of the generated motion ($d_g(t)$).

C.4 Step 3: Definition of the Structural Model in Abaqus

C.4.1 Geometric and Material Description

For this case study, a structure with a simple geometry has been intentionally selected, whose characteristics can be easily reproduced. The model consists of a 15-story reinforced concrete building with a square plan of 20×20 m. The height of each story is 3 m, resulting in a total height of 45 m.

The floor system consists of solid concrete slabs 15 cm thick at all levels. The vertical structure is composed of square columns with a constant section of 70×70 cm, arranged in a regular grid with a spacing of 5 m in both directions. This arrangement forms a moment-resisting frame system.

To introduce torsional asymmetry and provoke a three-dimensional response under a unidirectional excitation, an eccentric elevator core has been incorporated. This core consists of two adjacent shafts of 3×2 m each, constructed with reinforced concrete walls of a constant thickness of 35 cm.

A distributed mass of $400[\text{kg}/\text{m}^2]$ has been placed on the surface of all slabs to represent the dead and live loads that will act on the building.

The material properties used for all elements are those of a linear elastic concrete with a Young's Modulus $E = 31.753$ GPa, a Poisson's ratio $\nu = 0.2$, and a density $\rho = 2400$ kg/m³.

C.4.2 Finite Element Mesh

The discretization of the model leads to a finite element mesh composed of a total of 660 nodes and 885 elements. These elements are divided into two types, according to Abaqus nomenclature: 360 **B31** beam elements (2-node linear elements) to model the columns, and 525 **S4R** shell elements (4-node quadrilateral elements with reduced integration) to model both the floor slabs and the elevator core walls.

A coarse mesh has been intentionally chosen so that the model complies with the restrictions of the Abaqus Learning Edition, which can be downloaded and used for free¹, thus allowing the reproducibility of this case study by readers [1, 20].

Figure C.9 shows the finite element mesh of the model, where the columns, slabs, and elevator core can be seen.

¹The SIMULIA Abaqus Learning Edition can be downloaded from the official Dassault Systèmes website, after registration: <https://www.3ds.com/edu/education/students/solutions/abaqus-le>

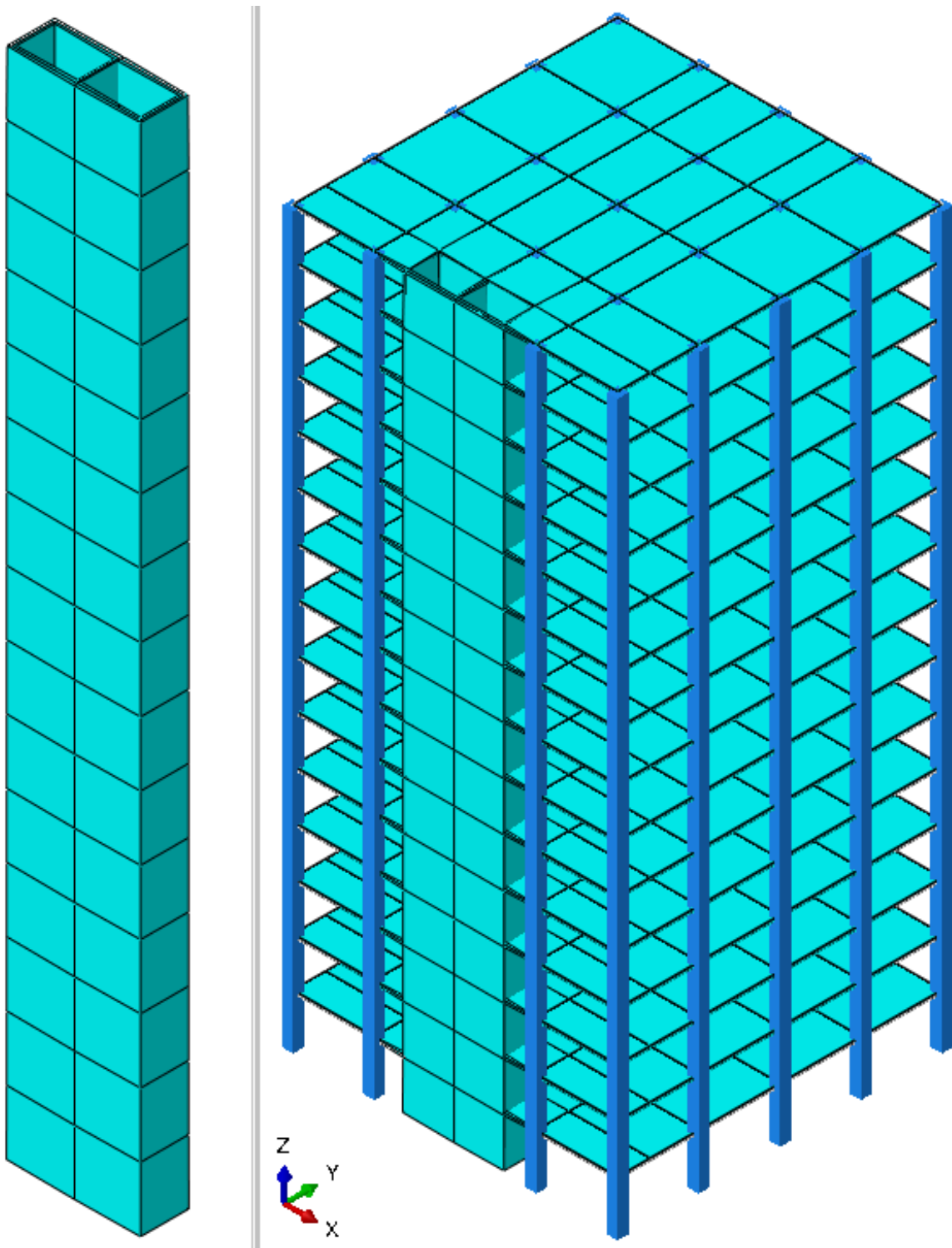


Figure C.9: View of the complete finite element model in Abaqus (right) and detail of the elevator wall (left).

C.4.3 Fundamental Dynamic Properties: Damping and Modes

Determination of Rayleigh Damping. The intrinsic damping of the system has been modeled using the classic **Rayleigh** model, which defines the damping matrix $[C]$ as a linear combination of the mass $[M]$ and stiffness $[K]$ matrices:

$$[C] = \alpha[M] + \beta[K] \quad (C.1)$$

The damping ratio for a vibration mode n with natural frequency ω_n is given by the expression:

$$\xi_n = \frac{\alpha}{2\omega_n} + \frac{\beta\omega_n}{2} \quad (C.2)$$

To determine the coefficients α and β , two pairs of anchor points, (ω_i, ξ_i) and (ω_j, ξ_j) , are defined. Substituting these two pairs of values into Equation C.2 generates a system of two linear equations with two unknowns (α and β). The solution to this system provides the unique values of α and β that ensure the damping curve passes exactly through the two defined anchor points [16]. For this study, the coefficients were calculated as $\alpha = 1.1875$ and $\beta = 0.00072222$, which were then implemented in the Abaqus model. Figure C.10 illustrates the resulting damping curve within the structure's frequency range, as well as the two anchor points used for its calibration.

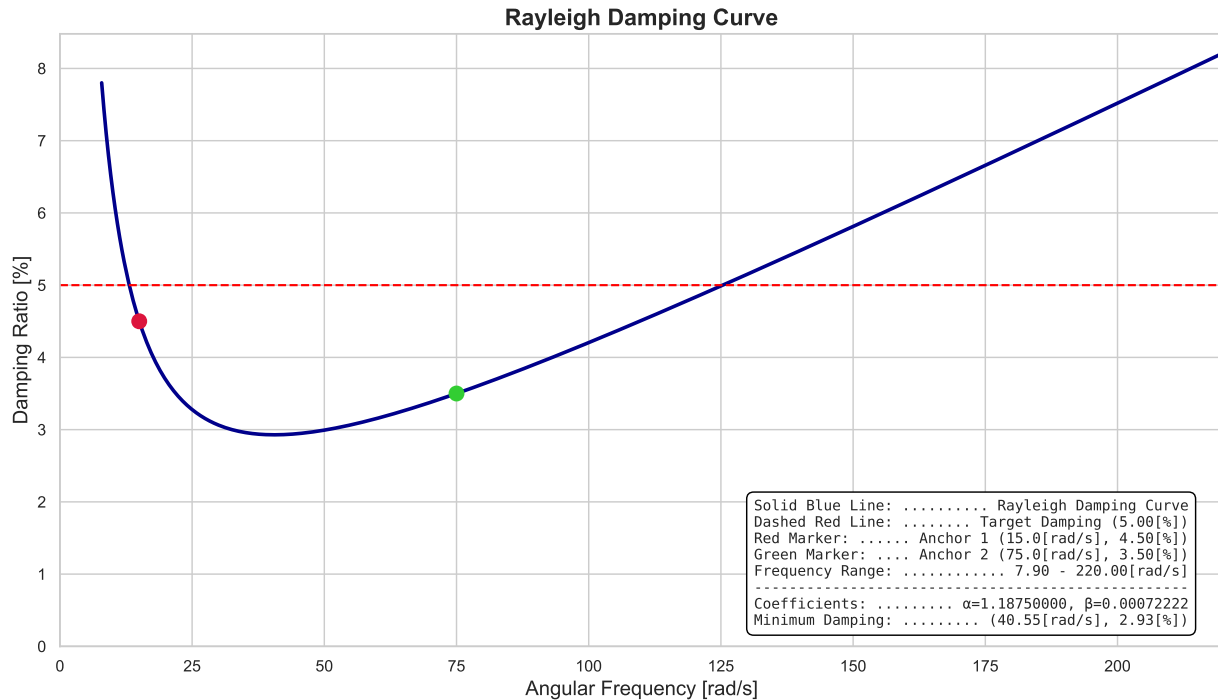


Figure C.10: Rayleigh damping curve defined for the structural model. The red and green points indicate the anchors used to calculate the coefficients α and β .

Frequencies, Periods, and Mode Shapes. The eigenvalue analysis determines the fundamental dynamic properties of the building. For this study, the first 50 modes of vibration have been extracted, which ensures the participation of more than 97% of the total mass in each principal horizontal direction, as shown in Figure C.11.

Total mass of model: 6373.68 tons

MODE	FREQUENCY		PERIOD
#	[rad/s]	[cycles/s]	[s]
1	7.9022	1.2577	0.795
2	9.9538	1.5842	0.631
3	13.272	2.1123	0.473
4	28.916	4.6022	0.217
5	32.398	5.1562	0.194
6	46.431	7.3898	0.135
7	57.056	9.0807	0.110
8	59.799	9.5173	0.105
9	62.077	9.8798	0.101
10	63.924	10.174	0.098

Total effective mass (50 modes):
X=6228.9, Y=6248.1, Z=5738.4
X=97.77%, Y=98.03%, Z=90.03%

Figure C.11: Frequencies of the first 10 modes and total participating masses.

The first four mode shapes, which usually dominate the seismic response, are presented in Figures C.12 to C.15. Each figure shows an isometric view and a plan view of the modal deformed shape, with a scale factor of 400 for better visualization.

- **Mode 1** ($T_1 = 0.795$ s): This is a pure flexural mode in the X direction, the most flexible direction of the building (see Figure C.12).
- **Mode 2** ($T_2 = 0.631$ s): This is a pure flexural mode in the Y direction (see Figure C.13).
- **Mode 3** ($T_3 = 0.473$ s): This is the first torsional mode. The eccentric inclusion of the elevator core introduces an asymmetry in the stiffness distribution, causing this rotational behavior that can be critical under seismic excitation (see Figure C.14).
- **Mode 4** ($T_4 = 0.217$ s): This corresponds to the second flexural mode in the X direction (see Figure C.15).

The clear separation between the periods of the translational modes (1 and 2) and the first torsional mode (3) is an indicator of an acceptable conceptual design, although the existence of torsion (intentionally created) will be fundamental to the overall response.

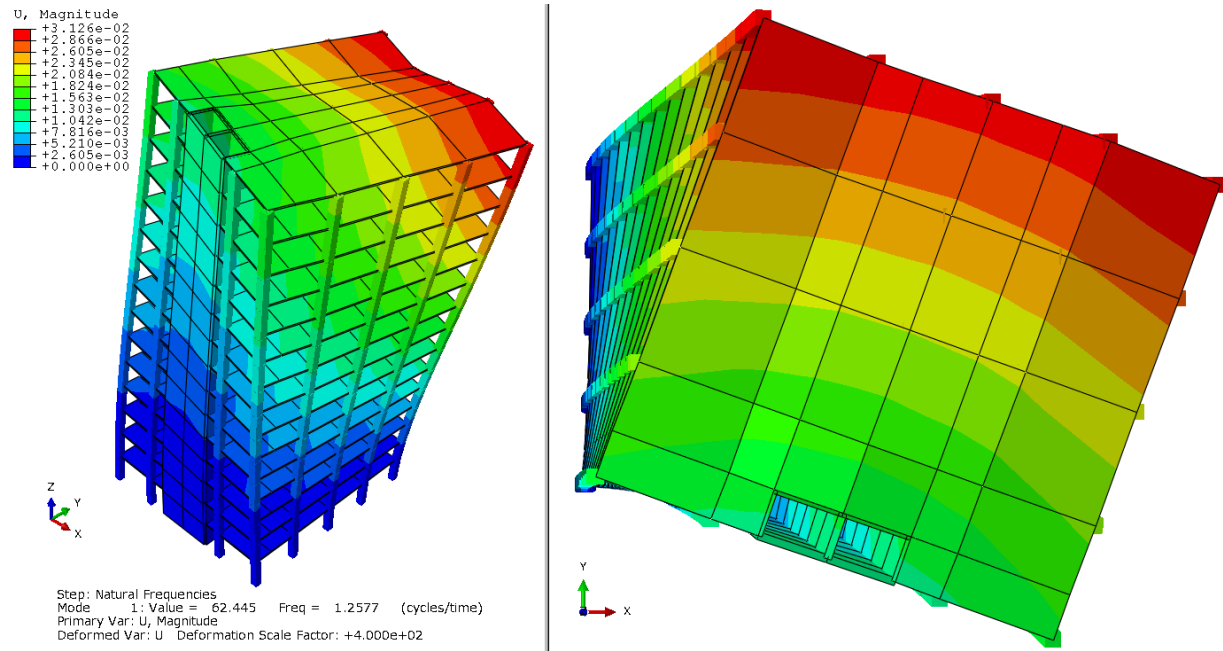


Figure C.12: Shape of the first mode of vibration (Flexion in X direction), $T_1 = 0.795$ s.

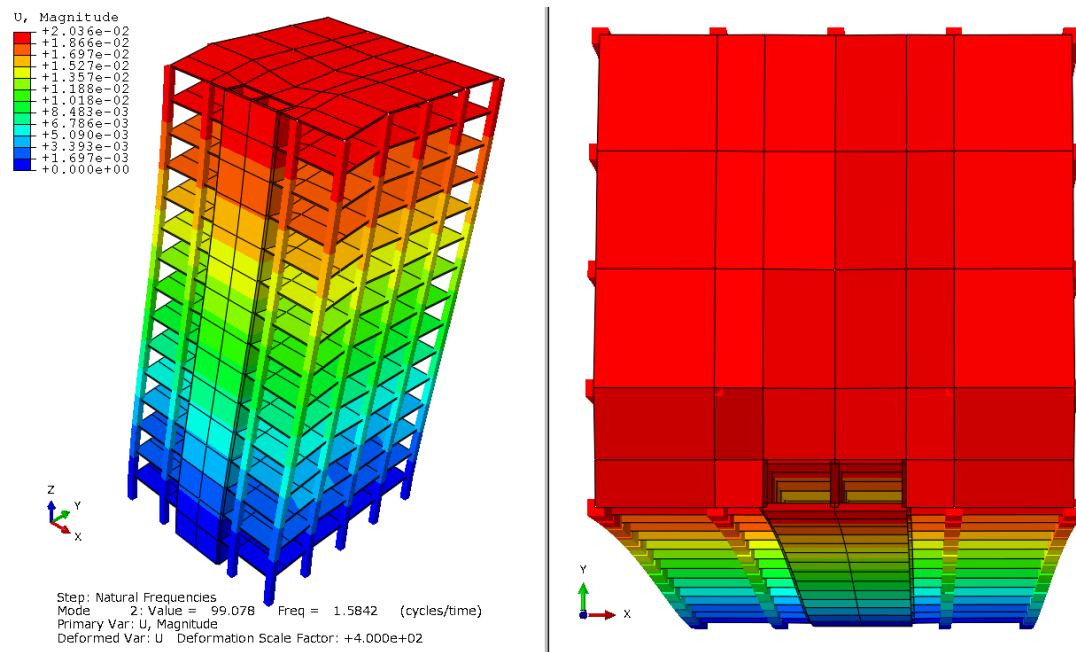


Figure C.13: Shape of the second mode of vibration (Flexion in Y direction), $T_2 = 0.631$ s.

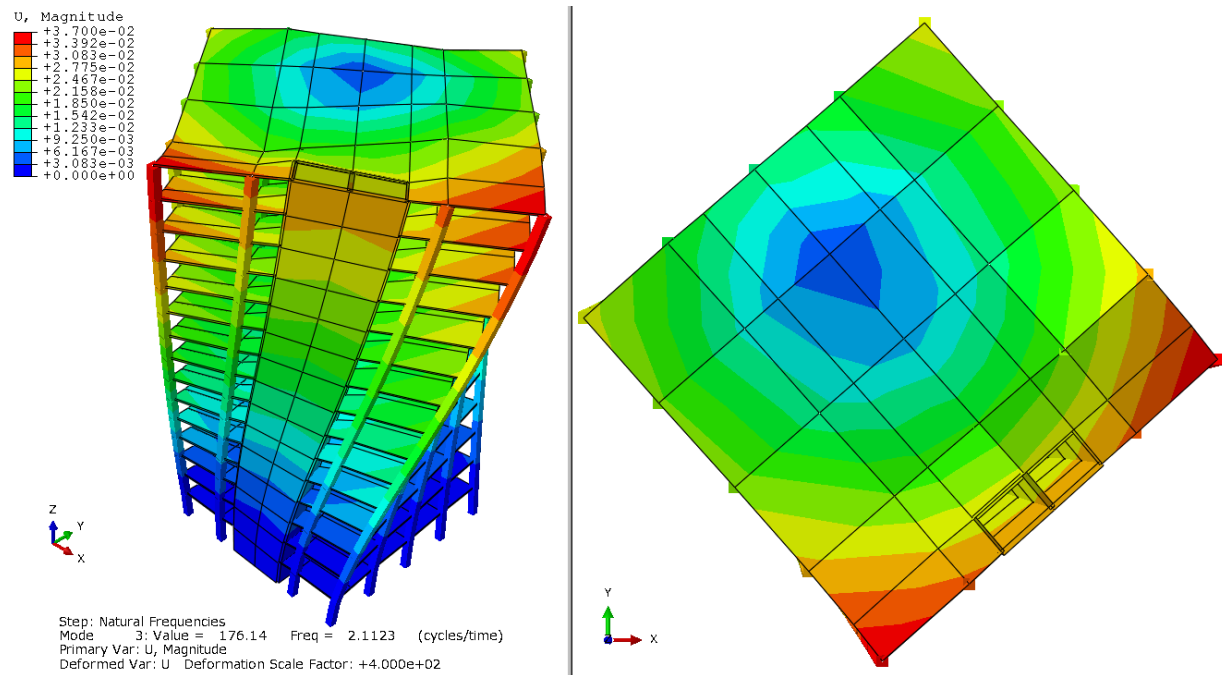


Figure C.14: Shape of the third mode of vibration (Torsion), $T_3 = 0.473$ s.

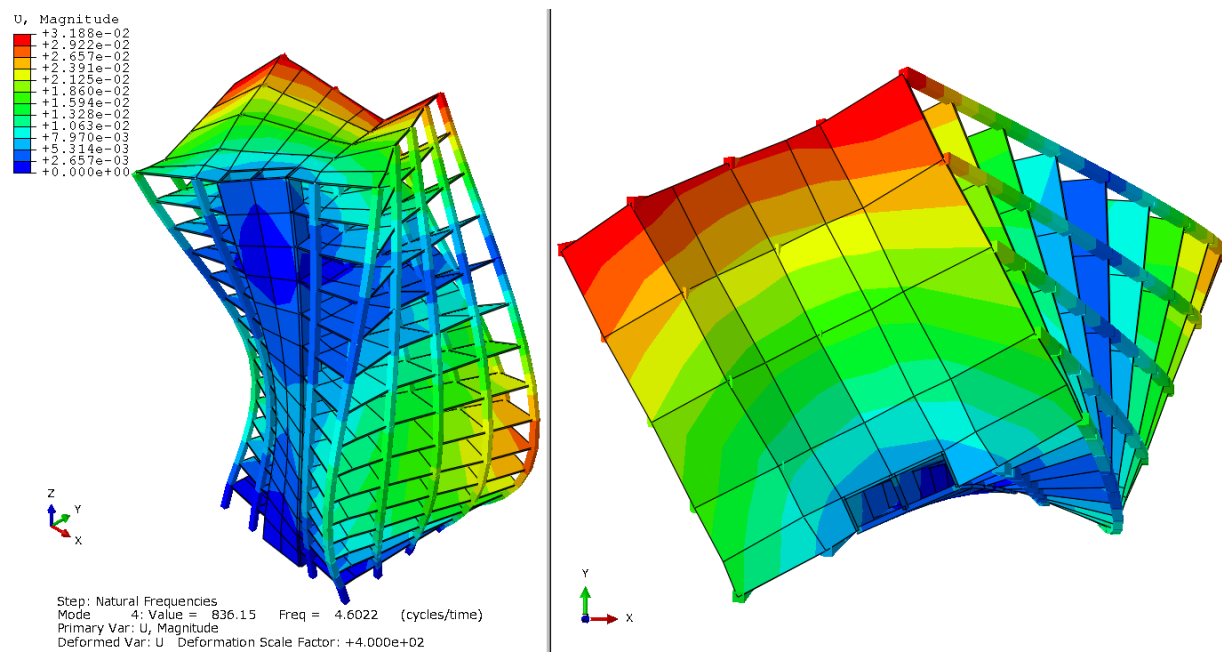


Figure C.15: Shape of the fourth mode of vibration (Second flexion mode in X), $T_4 = 0.217$ s.

C.5 Step 4: Execution and Comparison of Analysis Results

In this section, the four types of linear analysis are run on the Abaqus model, applying the generated artificial earthquake in the X direction, to compare and validate their results.

C.5.1 Results of the Modal Response Spectrum Analysis (MRSA)

In this analysis, the design spectrum from Figure C.1 is applied directly to the model in the X direction. The maximum response for each mode of vibration is calculated and then statistically combined (using the CQC method) to obtain an estimate of the total maximum structural response. Figure C.16 shows the resulting von Mises stress contour (from a fictitious, non-existent combined state), indicating a maximum stress of 30.73 MPa at the base of one of the first-level columns.

C.5.2 Results of the Time-Domain Analyses (MTHA and IDIA)

Next, the first two time-domain methods are applied using the complete displacement history generated by `earthquake_2025` (Figure C.8) as the load at the base of the model in the X direction. The Modal Time-History Analysis (MTHA) and the Implicit Direct-Integration Analysis (IDIA) are executed to obtain the complete time history of the response and compare their results.

The final validation consists of comparing the maximum response obtained. Figure C.17 superimposes the time history of the maximum von Mises stresses (calculated for all integration points in the model) obtained from both the IDIA (blue line) and MTHA (red line) analyses.

C.5.3 Results of the Time-Domain Analyses (IDIA and EDIA)

Additionally, to complete the cross-validation of the direct integration methods, a comparison is made between the Implicit (IDIA) and Explicit (EDIA) analyses. Both methods solve the complete equation of motion without resorting to modal decomposition. Figure C.18 shows the time history of the maximum von Mises stresses for both analyses, again applying the displacement history as the load at the base.

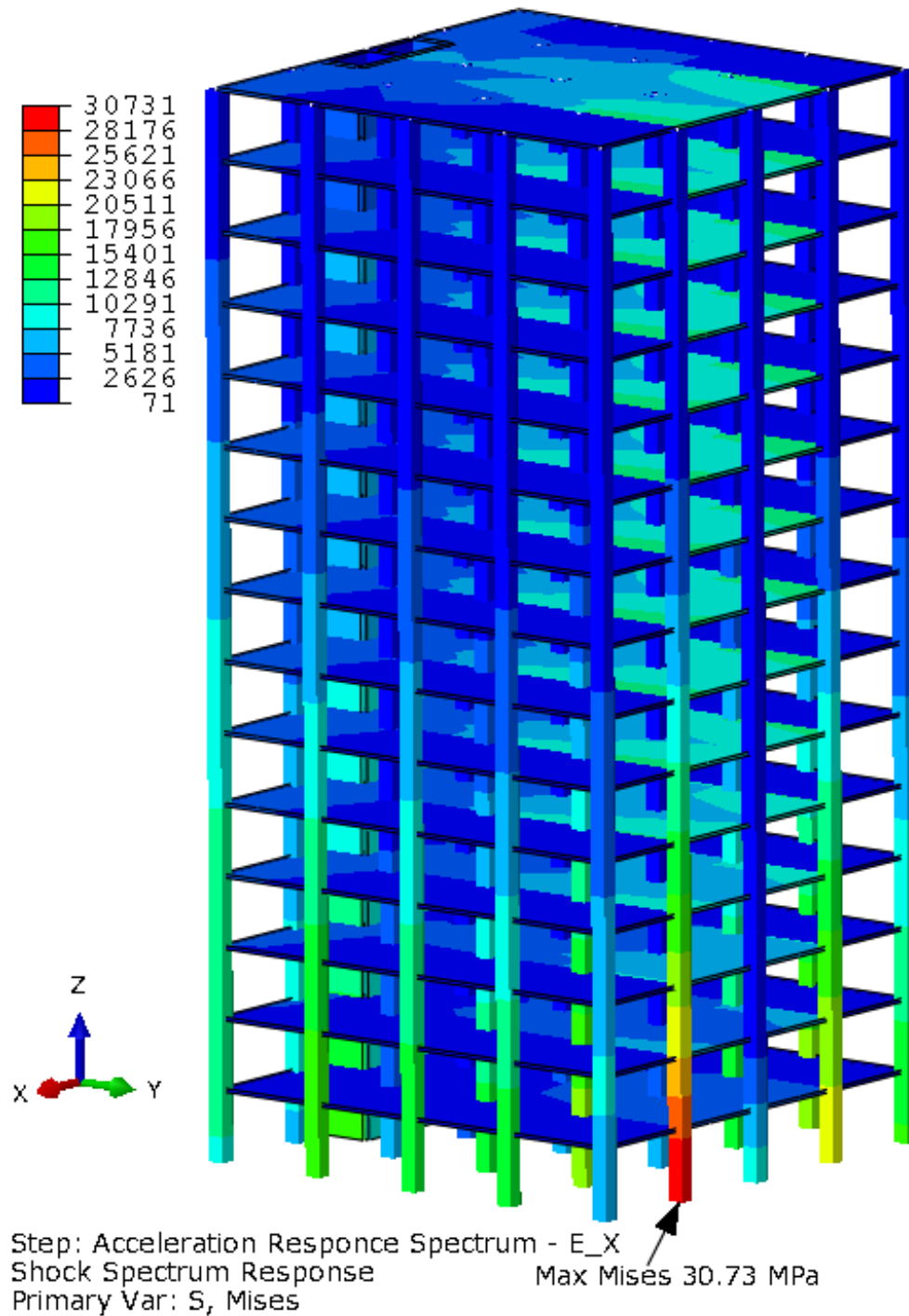


Figure C.16: Von Mises stress contour obtained from the Modal Response Spectrum Analysis (MRSA).

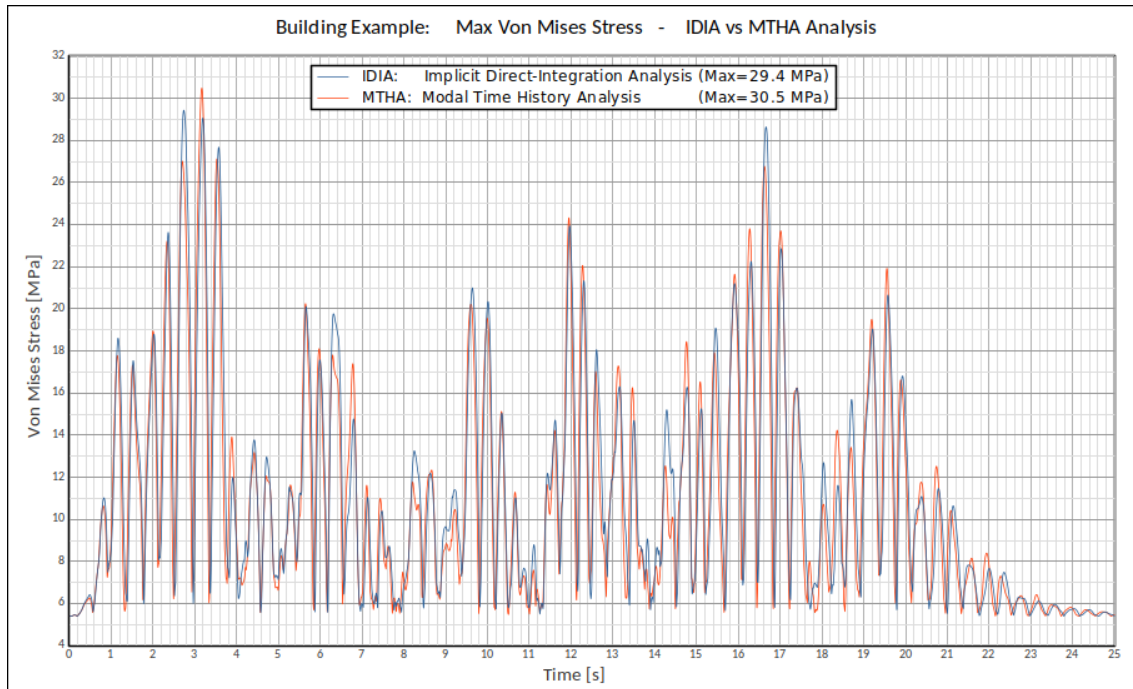


Figure C.17: Comparison of the time history of the maximum von Mises stress, which occurs at some point in the model at each instant, in the IDIA (blue line) and MTHA (red line) analyses.

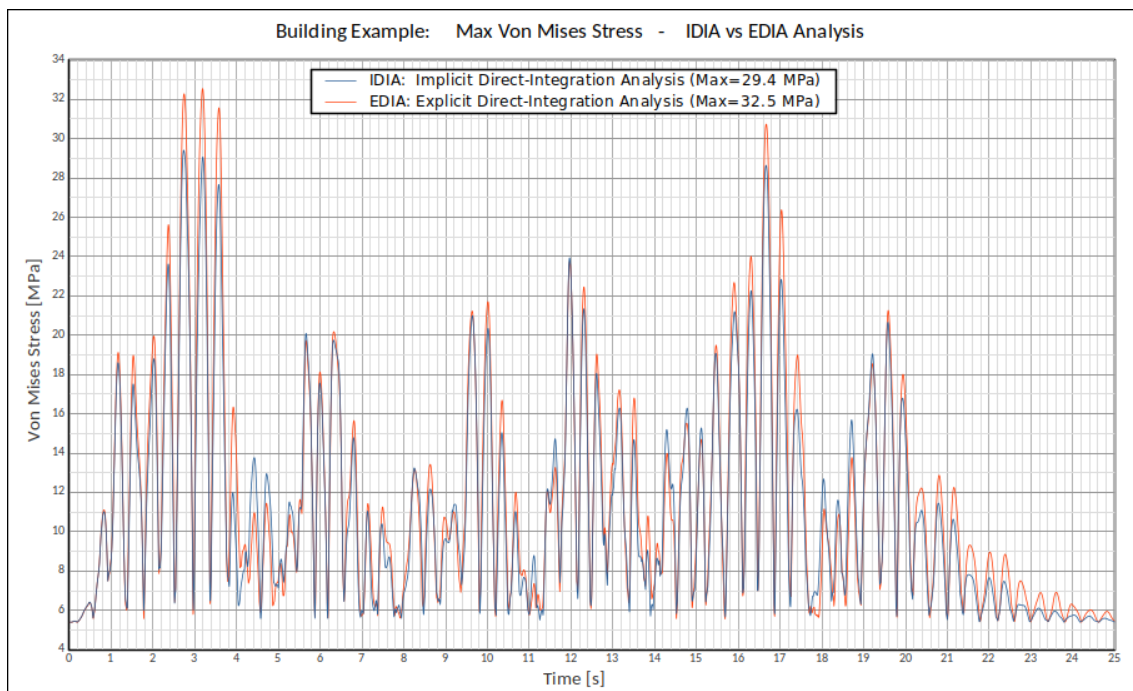


Figure C.18: Comparison of the time history of the maximum von Mises stress, which occurs at some point in the model at each instant, in the IDIA (blue line) and EDIA (red line) analyses.

C.5.4 Discussion and Conclusions of the Case Study

Note that all four analyses (MTHA, MRSA, EDIA, and IDIA) predict a very similar maximum von Mises stress. This maximum stress also occurs, for all four analyses, at the same point at the base of the same column.

An excellent correspondence is observed between the methods:

- The MRSA analysis predicts a maximum stress of 30.7 MPa.
- The MTHA analysis predicts a maximum stress of 30.1 MPa.
- The IDIA analysis predicts a maximum stress of 29.4 MPa.
- The EDIA analysis predicts a maximum stress of 32.5 MPa.

The small difference between the first three methods is expected and is due, on one hand, to the statistical nature of the MRSA's modal combination method (CQC) and the difference in precision between MTHA and IDIA. It is interesting to note that both exhibit similar numerical damping.

The difference in the maximum stress predicted by the direct integration methods—32.5 MPa for the explicit (EDIA) versus 29.4 MPa for the implicit (IDIA)—which occurs during the strong shaking phase of the earthquake, is a direct consequence of the inherent properties of each algorithm, known as **numerical damping**.

The implicit method (IDIA) possesses numerical damping that dissipates a small amount of energy in each time step, in addition to the physical damping defined by the Rayleigh model. This algorithmic dissipation accumulates throughout the simulation, resulting in a slightly more attenuated global response and, therefore, lower stress peaks. Conversely, the explicit central difference method (EDIA) is, by its formulation, non-dissipative; it does not introduce this artificial damping, so it tends to conserve the system's energy more accurately and produce slightly higher response peaks.

While the difference in the peak maximum occurs during the seismic excitation, this distinct behavior is most clearly visualized in the free vibration phase of the analysis (for $t > 20$ s), as shown in the detail of Figure C.19. In this section, free from external excitation, it is clearly observed how the response of the implicit analysis (blue line) decays more rapidly than that of the explicit analysis (red line), confirming the presence of greater total damping (physical + numerical) in the implicit scheme.

Despite these subtle differences, characteristic of each numerical formulation, the general shape of the response is practically identical in all cases, and the magnitude of the maximum is of the same order. The concordance in the magnitude of the maximum response is remarkable and conclusively validates the entire methodology. It demonstrates that the histories of acceleration, velocity, and displacement generated by `earthquake_2025` contain the necessary energy level and frequency demand to induce in the structure the same maximum response as the original design spectrum. This confirms that the program is a robust and reliable tool for creating artificial earthquakes for advanced structural analyses.

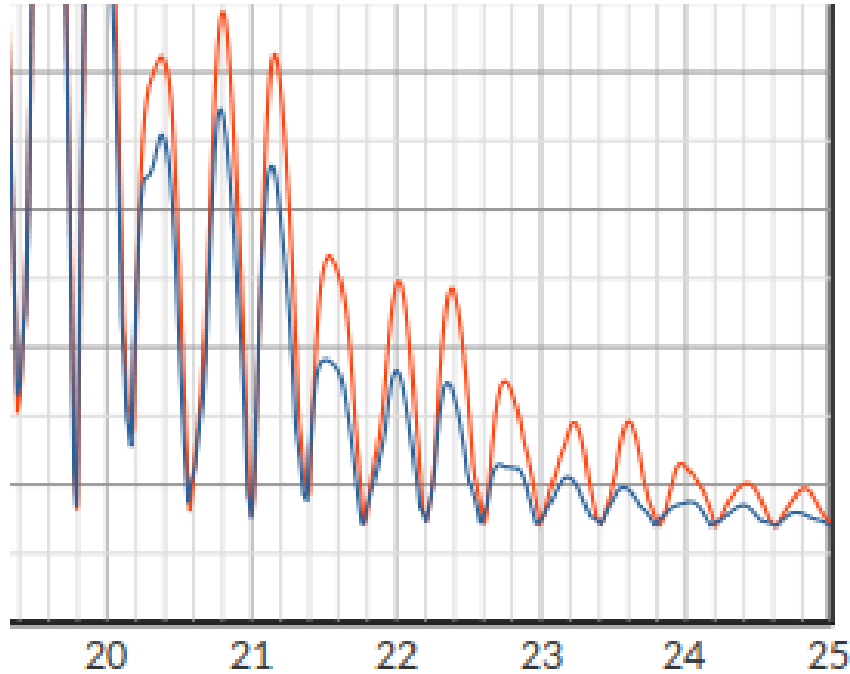


Figure C.19: Detail of the free vibration response ($t > 20$ s) from Figure C.18. The lower numerical damping of the explicit method (EDIA, red line) compared to the implicit one (IDIA, blue line) is evident.

This example reaffirms the discussion from Chapter 7: *for a linear problem, all methods lead, in practice, to the same response.* However, the moment nonlinearity is introduced (material plasticity, large deformations, contact, etc.), both MRSA and MTHA cease to be valid. In that scenario, Direct Integration Analysis, whether Implicit (IDIA) or Explicit (EDIA), fed by a realistic and compatible displacement history (an acceleration or velocity history can also be used) generated by a tool like `earthquake_2025`, becomes the only rigorous path for a reliable seismic evaluation.

Appendix D

Glossary of Terms and Symbols

This appendix provides a quick reference for the main acronyms, technical terms, and mathematical symbols used throughout this monograph.

D.1 Glossary of Acronyms and Key Terms

Acronym/Term	Definition
ADAS	Added Damping and Stiffness. A type of hysteretic energy dissipator that uses X-shaped metallic plates to dissipate energy through yielding.
ASCE	American Society of Civil Engineers. A U.S. organization that publishes reference standards in civil engineering, such as ASCE 7.
BSL	Building Standard Law. The building code of Japan, which establishes the minimum requirements for seismic design.
CAV	Cumulative Absolute Velocity. A measure of seismic intensity defined as the integral of the absolute value of acceleration over time. It is a robust indicator of the onset of structural damage.
CFE-MDOC	Comisión Federal de Electricidad - Manual de Diseño de Obras Civiles. A reference code document for seismic design in Mexico.
CP	Collapse Prevention. A structural performance level in which the building suffers severe damage but remains standing, allowing for evacuation and preventing collapse.
CQC	Complete Quadratic Combination. A modal combination rule that accounts for the correlation between modes with closely spaced frequencies. It is more rigorous than the SRSS rule.
DBE	Design Basis Earthquake. A seismic hazard level with a low probability of exceedance (typically 10% in 50 years), for which the structure is expected to suffer repairable damage but maintain life safety.

Acronym/Term	Definition
Ductility	The ability of a material, section, or structure to undergo large inelastic deformations without a significant loss of strength. It is a fundamental property for seismic energy dissipation.
earthquake_2025	An open-source program for the generation of artificial seismic ground motion time histories (accelerograms, velocigrams, and seismograms) that are “super-compatible” with a target elastic response spectrum, using a two-phase hybrid methodology.
EDIA	Explicit Direct-Integration Analysis. A numerical method that solves the equation of motion step-by-step without the need to invert a stiffness matrix, but it is conditionally stable.
EDP	Engineering Demand Parameter. A physical quantity calculated through structural analysis (e.g., interstory drifts, plastic rotations) that correlates with the level of damage.
EPSD	Evolutionary Power Spectral Density. An extension of the PSDF that allows the frequency content of a signal to vary over time, modeling non-stationarity.
Eurocode 8	The European standard for the design of structures for earthquake resistance (EN 1998).
FAS	Fourier Amplitude Spectrum. A representation of the amplitude of the frequency components of a signal, obtained from the Fourier Transform.
FPS	Friction Pendulum System. A type of seismic base isolator whose period is defined by the radius of curvature of a concave surface and which dissipates energy through friction.
GMPE	Ground Motion Prediction Equations. Empirical relationships, based on data from real earthquakes, that predict the level of ground motion (e.g., PGA, $S_a(T)$) at a site for an earthquake of a given magnitude and distance.
GUI	Graphical User Interface. A visual environment that allows for more intuitive interaction with a software.
Hanning Window	A mathematical function used in signal processing to smooth the ends of a signal segment, forcing it to start and end at zero continuously. In earthquake_2025 , it is used to modulate the corrective <i>wavelets</i> , thus preventing the introduction of discontinuities in the accelerogram.
HDR	High Damping Rubber Bearing. An elastomeric seismic isolator made with a rubber compound of high intrinsic damping.

Acronym/Term	Definition
HI	Housner Intensity. A measure of structural damage potential, calculated as the integral of the pseudo-velocity spectrum (for $\xi = 5\%$) over the period range of 0.1 to 2.5 seconds.
Hysteresis	The nonlinear behavior of a system in which the response to a load depends on its past history. On a force-displacement diagram, it manifests as loops whose area represents the dissipated energy.
ID	Destructive Potential Factor. An intensity measure sensitive to the asymmetry of the seismic motion, calculated from the integral of the squared acceleration weighted by the sign of the velocity. <code>earthquake_2025</code> reports its positive (I_D^+) and negative (I_D^-) components separately.
IDIA	Implicit Direct-Integration Analysis. A robust numerical method for solving the equation of motion (linear or nonlinear) that requires solving a system of equations at each time step.
IO	Immediate Occupancy. A structural performance level in which the building suffers minimal damage and can be reoccupied almost immediately after the earthquake.
LRB	Lead Rubber Bearing. An elastomeric seismic isolator that incorporates a central lead core to significantly increase hysteretic energy dissipation.
LS	Life Safety. A structural performance level in which the building suffers significant damage but collapse is prevented, ensuring the safety of the occupants.
MCE	Maximum Considered Earthquake. The most severe level of seismic hazard that a structure must resist without collapsing, usually with a very low probability of exceedance (e.g., 2% in 50 years).
MDOF	Multiple-Degree-of-Freedom System. A structural model that requires multiple coordinates to describe its motion.
MMI	Modified Mercalli Intensity. A qualitative scale that describes the severity of an earthquake at a specific location based on observed effects.
MRSA	Modal Response Spectrum Analysis. A seismic analysis method based on the superposition of the maximum responses of each mode of vibration, obtained from a design spectrum.
MTHA	Modal Time-History Analysis. An analysis method that decomposes the response into vibration modes and solves the response of each in time, then superimposes them.
NBCC	National Building Code of Canada.

Acronym/Term	Definition
NBDS-2023	Norma Boliviana de Diseño Sísmico 2023. The reference code document in Bolivia.
NCh433	Norma Chilena 433. The reference standard for the seismic design of buildings in Chile.
NLTHA	Non-Linear Time History Analysis. A structural analysis method that solves the nonlinear equation of motion step-by-step in time, using an accelerogram as excitation.
NTE E.030	Norma Técnica de Edificación E.030. The reference standard for earthquake-resistant design in Peru.
PBD	Performance-Based Design. A design philosophy that aims to achieve explicit performance objectives (e.g., Immediate Occupancy, Life Safety) for different levels of seismic hazard.
PFA	Peak Floor Acceleration. The maximum absolute acceleration experienced within a structure. It is a critical metric for the design of non-structural components.
PGA	Peak Ground Acceleration. The absolute maximum value of acceleration recorded during an earthquake.
PGD	Peak Ground Displacement. The absolute maximum value of ground displacement.
PGV	Peak Ground Velocity. The absolute maximum value of ground velocity.
PSA	Pseudo-Acceleration. The response spectrum ordinate that represents the maximum elastic force per unit mass in an SDOF.
PSDF	Power Spectral Density Function. A function that describes how the power (variance) of a stochastic process is distributed in the frequency domain.
PSHA	Probabilistic Seismic Hazard Analysis. A methodology that integrates information from all seismic sources to calculate the probability of exceeding a certain level of motion at a site.
PSV	Pseudo-Velocity. The response spectrum ordinate related to the maximum strain energy in an SDOF.
SASID	Sistema de Acciones Sísmicas de Diseño. An online tool by CFE that provides seismic design parameters (such as a_0, c, T_a, T_b) for any point in Mexican territory.
SD	Spectral Displacement. The response spectrum ordinate that represents the maximum relative displacement in an SDOF.

Acronym/Term	Definition
SDOF	Single-Degree-of-Freedom System. An idealized structural model with a single coordinate to describe its motion.
SLE	Service Level Earthquake. A frequent earthquake with a high probability of occurrence, for which the structure is expected to remain essentially undamaged.
Spectral Sweep	An algorithmic technique used in Phase 2 of earthquake_2025 , consisting of iteratively passing through all periods of the response spectrum to attempt pointwise corrections using <i>wavelets</i> .
SRSS	Square Root of the Sum of Squares. A modal combination rule that assumes the maximum modal responses are statistically independent.
SSI	Soil-Structure Interaction. An advanced analysis that considers the coupled behavior of the structure and the foundation soil.
Super-Compatibility	A term coined to describe the exceptionally high level of fit between the response spectrum of a generated artificial earthquake and the target spectrum, achieved through the two-phase methodology of the earthquake_2025 program.
TADAS	Triangular Added Damping and Stiffness. A type of hysteretic energy dissipator that uses triangular-shaped metallic plates.
TMD	Tuned Mass Damper. A passive control device added to a structure to reduce its vibrations by tuning its frequency to that of the structure.
UHS	Uniform Hazard Spectrum. A response spectrum in which all spectral ordinates have the same annual probability of being exceeded.
USGS	U.S. Geological Survey. A scientific agency of the United States government that provides seismic hazard data and maps.
Wavelet (Corrective)	A small, time-localized wave signal (a sinusoid modulated by a Hanning window) that is added to the accelerogram during Phase 2 to correct the spectral difference at a specific period with minimal disturbance to the overall signal.

D.2 Nomenclature and Symbols Used

Symbol	Description	Units
$a_g(t)$	Ground acceleration time history.	L/T ² (e.g., m/s ²)
a_0	Ordinate at the origin of the spectrum (Mexico Code).	L/T ²
a_{gR}	Reference peak ground acceleration on rock (Eurocode 8).	g or L/T ²
A	Area of the fault surface that slips.	L ²
$[A], [B]$	State transition and load application matrices in numerical integration.	-
A_i	Amplitude of the i-th sinusoid in accelerogram synthesis.	L/T ²
A_0	Maximum effective acceleration on rock (Chilean Code).	g
c	Viscous damping coefficient of an SDOF.	F·T/L
c	Plateau ordinate of the spectrum (Mexico Code).	L/T ²
C	Seismic amplification factor (Peruvian Code).	-
$[C]$	Damping matrix of the MDOF system.	F·T/L
C_0	Standard base shear coefficient (Japanese Code).	-
d_i	Normalized difference at spectral point i .	%
$\bar{d}$	Mean of the normalized differences.	%
d_{eq}	Weighted equivalent difference.	%
d_{T5}	Average of the top 5% of normalized differences.	%
D	Average displacement on the fault.	L
D_{5-95}	Significant duration (5%-95% interval of AI).	T
E	Young's Modulus (Modulus of elasticity).	F/L ² (e.g., GPa)
E_s	Maximum elastic strain energy in an SDOF.	F·L
F_a, F_v	Site factors for short and long periods (ASCE 7 and NBDS Codes).	-
$\{F_{eff}\}$	Effective force vector in implicit integration.	F
$\{F_R\}$	Vector of internal resisting forces of the system.	F
f_s	Internal elastic force in an SDOF.	F
g	Acceleration of gravity.	L/T ²
G	Shear modulus of the material.	F/L ²

Symbol	Description	Units
$G(\omega)$	Power Spectral Density Function.	$(\text{L}/\text{T}^2)^2/(\text{rad}/\text{T})$
H	Thickness of a soil layer.	L
$H(t)$	Husid plot. Integral of the squared acceleration over time.	L^2/T^3
HI	Housner's Spectral Intensity.	L
$\{I\}$	Influence vector of the ground motion.	-
$I(t)$	Intensity envelope function.	-
I_A	Arias Intensity.	L/T (e.g., m/s)
I_D, I_D^+, I_D^-	Destructive Potential Impulse (total, positive, and negative).	L/T
I_E	Importance factor of the structure.	-
I_s	Seismic impedance of a material (ρV_S).	$\text{F} \cdot \text{T}/\text{L}^3$
k	Elastic stiffness of an SDOF.	F/L
$[K]$	Stiffness matrix of the MDOF system.	F/L
$[K_{eff}]$	Effective stiffness matrix in implicit integration.	F/L
$[K_T]$	Tangent stiffness matrix of the nonlinear system.	F/L
m	Mass of an SDOF.	$\text{F} \cdot \text{T}^2/\text{L}$
$[M]$	Mass matrix of the MDOF system.	$\text{F} \cdot \text{T}^2/\text{L}$
M_0	Seismic moment.	$\text{F} \cdot \text{L}$
M_W	Moment Magnitude.	-
N	Total number of points or components in a sum.	-
$\{P(t)\}$	Vector of applied external forces.	F
P_{95}	95th percentile of the normalized differences.	%
q	Behavior factor (Eurocode 8).	-
$q_n(t)$	Coordinate of mode n in time.	L
r	Decay exponent of the spectrum (Mexico Code).	-
R	Response modification factor.	-
S	Soil amplification factor (Eurocode 8, NTE E.030).	-
S_0	Peak ground acceleration on rock (Bolivian Code).	g
$S_a(T)$	Pseudo-acceleration spectrum ordinate at period T.	g or L/T^2
$S_D(T), SD$	Displacement spectrum ordinate at period T.	L
$S_V(T), PSV$	Pseudo-velocity spectrum ordinate at period T.	L/T

Symbol	Description	Units
S_{DS}, S_{D1}	Design spectral parameters (ASCE 7).	g
S_S, S_1	Spectral parameters on rock for short period and 1 second (ASCE 7).	g
T, T_n	Natural period of vibration.	T (e.g., s)
T_a, T_b	Start and end periods of the plateau (Mexico Code).	T
T_B, T_C, T_D	Characteristic periods of the spectrum (Eurocode 8).	T
T_d	Total duration of a seismic record.	T
T_L	Long-period transition period (ASCE 7).	T
T_{min}	Period of the highest vibration mode of an FE mesh.	T
T_P, T_L	Characteristic periods of the spectrum (Peruvian Code).	T
T_s	Fundamental period of a soil deposit.	T
T_S, T_0	Characteristic periods of the spectrum (ASCE 7).	T
T'	End period of the plateau (Chilean Code).	T
$u(t)$	Relative displacement of an SDOF.	L
$\dot{u}(t), \ddot{u}(t)$	Relative velocity and acceleration of an SDOF.	L/T, L/T ²
$\{u(t)\}$	Displacement vector of an MDOF.	L
V_P, V_S	Velocity of P- and S-waves.	L/T
$V_{S,30}$	Average shear wave velocity in the top 30 m.	L/T
Z	Seismic zone factor.	-
α, β	Rayleigh damping coefficients.	T, 1/T
$\alpha(T)$	Dynamic amplification factor (Chilean Code).	-
β, γ	Newmark's integration parameters.	-
Γ_n	Participation factor of mode n.	-
δ_u, δ_y	Ultimate and yield displacement.	L
Δt	Time step in numerical integration.	T
Δt_{crit}	Critical time step for stability of the explicit method.	T
$\Delta \omega_i$	Frequency band width of component i .	rad/T
ζ_p	Peak factor for the response of an SDOF.	-
λ	Wavelength.	L
μ	Ductility factor.	-

Symbol	Description	Units
μ	Rock rigidity modulus (in the context of M_0).	F/L ²
ν	Poisson's ratio.	-
ξ	Viscous damping ratio (with respect to critical).	-
π	Mathematical constant Pi ($\approx 3.14159\dots$).	-
ρ	Mass density of the material.	F·T ² /L ⁴
ρ_{ij}	Modal correlation coefficient (for CQC).	-
σ^2	Variance of a stochastic process.	(signal unit) ²
σ_d	Standard deviation of the normalized differences.	%
ϕ_i	Random phase angle of the i-th sinusoid.	rad
$\{\phi_n\}$	Shape of vibration mode n.	-
ω, ω_n	(Undamped) natural circular frequency.	rad/T

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